PHYSICAL REVIEW
LETTERS

VOLUME 1 NOVEMBER 1, 1958 NUMBER 9

OPTICAL DETECTION OF NARROW Rb^{87} HYPERFINE ABSORPTION LINES

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(Received October 8, 1958)

Within the last year optical detection of the ($M_F=0 \rightarrow M_F=0$) ground state hyperfine transition has been reported for sodium vapor[1,2] and for cesium vapor.[3,4] The collisional narrowing technique suggested by Dicke[5] was used to reduce the Doppler width. Results have now been obtained for Rb^{87} vapor using a different optical pumping method. Absorption lines a factor 300 narrower than the normal Doppler width have been achieved.

The experimental arrangement is shown in Fig. 1. The sample consists of a Pyrex bulb containing Rb^{87} vapor and a suitable buffer gas. Light from a rubidium spectral lamp is passed through a filter bulb containing Rb^{85} vapor and then through the sample. The scattered light from the sample is monitored by a photomultiplier tube.

The strongest lines emitted by the rubidium lamp are the two resonance lines[6] at 7800 and 7947A. These lines are absorbed by the rubidium vapor in the filter and in the sample and cause transitions to the $5\,^2P_{3/2}$ and $5\,^2P_{1/2}$ excited states. The hyperfine components of the Rb^{85} lines are separated by only half as much as the Rb^{87} hyperfine components and have slightly different mean wavelengths. When about 5 cm of argon is added to the filter bulb, the Rb^{85} absorption lines are somewhat broadened. The lower energy hyperfine components of the Rb^{87} lines from the lamp are then absorbed considerably more strongly in the filter than the higher energy ones. The light reaching the sample thus causes transitions mainly from the lower ground state hyperfine level to the excited states. Since the atoms return by spontaneous emission to both ground-state levels, they tend to accumulate in the higher level.[7]

Under the conditions described above where the intensities of the hyperfine components of the incoming light are not equal, the intensity of the light scattered by the sample depends on the populations of the two levels. It thus provides a means of monitoring the relative populations. When an oscillating magnetic field having the right frequency to cause the ($F=2$, $M_F=0 \rightarrow F=1$, $M_F=0$) hyperfine transition is applied to the sample, the population difference between the two hyperfine levels decreases and the intensity

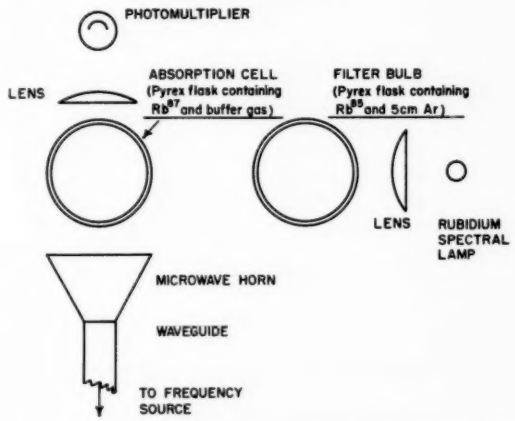

FIG. 1. Apparatus for observing the Rb^{87} hyperfine transition by an optical pumping method.

of the scattered light reaching the photomultiplier increases.

With the hyperfine filtering method described above measurements have been made of the resonance frequencies, line widths, and signal strengths for a number of sample bulbs containing various buffer gases. Most of the bulbs used were 500-ml Pyrex flasks which were thoroughly outgassed before filling. About 3 mg of Rb^{87} was distilled into each bulb, and then the desired buffer gas was introduced and the bulb sealed off. The rubidium vapor pressure of about 2×10^{-7} mm at room temperature gave an absorption coefficient of about unity for the bulbs with fairly low buffer gas pressures. The inert gases and mixtures used were of spectroscopic grade, while the other gases came from sources believed to be of high purity. An accelerated aging test gave no indication of a frequency shift for a bulb held at 200°C for one week.

In Table I the observed pressure and temperature shifts measured near room temperature are given. The temperature shifts are the observed shifts in frequency for a given change in temperature of the whole bulb, and thus are not measured at constant pressure. It is clear that, as was observed[4] for Cs^{133}, the pressure and temperature shifts are not simply related. However, the results given for the two mixtures demonstrate that it is possible to reduce greatly either the pressure or the temperature coefficient by choosing the proper composition. By using three gases together, such as neon, argon, and one of the hydrocarbons, it should be possible to reduce strongly both the pressure and temperature dependence. The accuracy of most of the pressure shifts given is limited almost entirely by uncertainty in the buffer gas pressures. The observed shifts are linear with pressure and extrapolate to the same zero-pressure frequency within the uncertainties in the measurements.

Table I. Changes in the hyperfine transition frequency for Rb^{87} absorption cells with pressure of the buffer gas and with temperature of the cell. The pressure coefficients were measured at constant temperature and the temperature coefficients at constant volume.

Buffer gas	Freq. shift in cps per cm of buffer gas pressure.	Temperature shift in cps per degree centigrade per cm of buffer gas pressure.
hydrogen	+6600±130	+10
deuterium	+6700±150	+10
helium	+7200±140	+10
neon	+3920± 80	+ 2.6
nitrogen	+5200±100	+ 6
argon	- 510± 10	- 3.0
krypton	-5800±500	- 5
methane[a]	-5000	- 6
n-pentane[a]	-28 000	- 7
neopentane[a]	-29 000	+ 2
n-heptane[a]	-42 000	+ 2
mixture I (11.7%Ne + 88.3%A)	+ 14 ±3	- 2.4
mixture II (50%Ne + 50%A)	+1700 ±40	- 0.2

[a] The values for the hydrocarbons are somewhat uncertain because of possible reactions with stopcock grease in the vacuum system and pyrolysis in sealing off the samples.

Line width and signal strength measurements were made with the light intensity and microwave power at levels which contributed little to the widths of the narrowest lines observed. The line width as a function of pressure gave a very broad minimum at several cm for most of the buffer gases used. For pressures above the minimum, the signal strength at room temperature went down quite rapidly, while the line width went up slowly. For neon and helium the line widths at 20 cm were about 25 and 40 cps, respectively.

The narrowest lines were obtained using neon and helium at pressures of several cm. The signal strength corresponded to a 5% change in the amount of light scattered from the Rb^{87} vapor. The line width was about 20 cycles, giving a Q of over 300 million.[8] For argon the best lines were about 30 cps wide. Strong and narrow lines were also obtained for some of the hydrocarbons at pressures of several mm.

The frequency source used in the experiments described above was provided by Mr. W. Levitt of the Radio Techniques Branch, National Research Laboratory. For most of the measurements the applied frequency was sinusoidally swept at 4 cps between the points of maximum and minimum slope of the line, and the photomultiplier output displayed with a 30-cps bandwidth on an oscilloscope. Although the frequency could be set on the center of the hyperfine absorption line to 5 parts in 10^{11}, the over-all accuracy of the frequency measurements was not quite this high because of random minute-to-minute variations of 1 or 2 parts in 10^{10} in the frequency source.

For mixture I a fairly good extrapolation to zero pressure could be made. The accuracy was limited to about 1 part in 10^9 by a scatter of several cps in the points obtained with different sample bulbs. A set of 100-ml sample bulbs was also used in this extrapolation to eliminate the possibility of a size effect. The frequency obtained was 6 834 682 608 ± 7 cps with respect to the A.1 time scale introduced by Markowitz[9] on which the Cs^{133} frequency is 9 192 631 770 cps. This is in agreement with an extrapolated value of 6 834 682 535 ± 70 cps on the same scale obtained by Carver by a power absorption method.[10] However, since a preliminary extrapolation for Cs^{133} vapor made in the same way disagreed with atomic beam results by considerably more than the known uncertainties,[4] there is still some doubt about the interpretation of the result.

We wish to express our thanks to Mr. W. Levitt and Mr. R. Stone of the Naval Research Laboratory Radio Techniques Branch for supplying the frequency source and synthesizer used in this work. Most of the sample bulbs used were filled by Mr. N. Williams of the National Bureau of Standards Atomic Physics Section. We have benefitted greatly from conversations with T. R. Carver, R. H. Dicke, A. L. Bloom, H. G. Dehmelt, C. O. Alley, M. Arditi, and W. E. Bell on optical pumping techniques.

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VALENCE BAND STRUCTURE OF SILICON

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Cyclotron resonance experiments have indicated the existence of two kinds of holes with different effective masses in germanium, as well as in silicon.[1] Theoretical calculations by Herman[2] have given a picture of the band structure of these crystals, according to which the valence band is split into three sub-bands shown schematically in Fig. 1. The spin-orbit interaction causes a splitting between the levels $p_{3/2}$ (bands 1 and 2) and $p_{1/2}$ (band 3). The degeneracy of the $p_{3/2}$ level is cancelled for crystal momentum $k \neq 0$ which gives rise to the sub-bands 1 and 2 with different curvature, i.e., with the different effective masses observed in cyclotron resonance. The sub-band 3 has too low a population of holes at moderately high temperatures to be observed.

Full evidence for this theoretically predicted structure has been obtained for Ge from the infrared absorption spectra of p-type germanium,[3,4] germanium with electrically injected holes,[5] and photoelectrically excited intrinsic germanium.[6,7] These spectra show distinct bands which can be unambiguously attributed to the three interband transitions 1-2, 2-3, and 1-3 with maxima at approximately 1000 cm^{-1}, 2100 cm^{-1}, and 3000 cm^{-1} at room temperature. By examining the

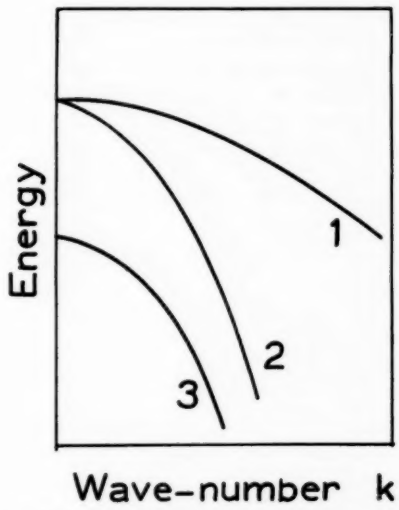

FIG. 1. Valence energy band contours of silicon and germanium.

temperature dependence of the maxima, one can get an approximate confirmation of the cyclotron resonance masses m_1 and m_2 and one can calculate, for the spin-orbit splitting in the origin, $E_1 - E_3 = 0.28$ ev.

Attempts to find the corresponding structure in p-type silicon have hitherto been unsuccessful.[4,8] Kahn[9] has derived formulas for the absorption coefficient for the three interband transitions, according to which the maximum of the 1-2 transition should lie at about 400 cm^{-1} at room temperature. From a comparison of the spin-orbit splitting in the atoms of Ge and Si, Kahn estimates the separation between the bands 1 and 3 in silicon to be about 300 cm^{-1} or 0.04 ev. In this spectral region silicon possesses an extended and complicated spectrum due to lattice vibrations.[4,10] Further, the masking effect due to intra-band free-hole absorption has been put forward as an explanation of the failure to find a structure analogous to that of Ge.[4]

By applying the technique of photogeneration of free carriers,[7] the present authors have now been able to excite and observe an absorption spectrum in silicon, which probably emanates from these expected valence interband transitions. The experimental arrangement was identical to that previously used for germanium.[7] A single crystal of p-type silicon, resistivity about 1000 ohm cm, polished and etched, dimensions 5×10×15 mm, was placed before the entrance slit of a Perkin Elmer Model 13 spectrophotometer equipped with cesium bromide optics and with an appropriate filter or a reststrahlen plate inserted into the beam. Illumination of the crystal with a tungsten ribbon lamp caused an increase in the absorption coefficient, the spectral distribution of which is shown in Fig. 2. Because, in

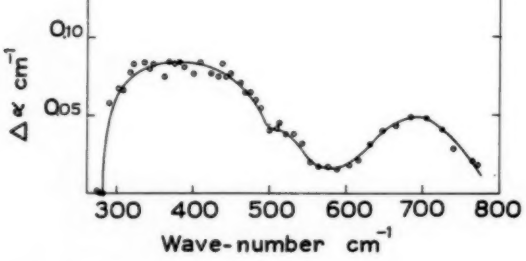

FIG. 2. Increase in the absorption coefficient, $\Delta\alpha$, of Si, due to illumination, as a function of wave-number.

this region, the absorption coefficient due to lattice vibrations fluctuates rapidly about a mean value $\alpha \sim 5$ cm^{-1}, it would be very difficult to detect differences $\Delta\alpha$ in the magnitude observed here by means of a comparison between different p-type specimens with different acceptor concentrations.

As well as the photoeffect, which took place instantaneously considering the time constant of the detector device, we could observe a further increase of the absorption coefficient due to longer illumination, apparently caused by the heating of the sample. This latter increase, however, must be disregarded because a temperature increase also affects the lattice absorption.

It should be evident that the spectrum obtained is due to free carriers, liberated by the photoelectric effect. In the present experiment, it is in principle impossible to decide whether the carriers are electrons or holes or both. The striking similarity to the hole absorption spectrum of germanium[3,5-7] leaves little doubt, however, that these are the predicted transitions between the valence sub-bands. Moreover, the position of the long-wavelength maximum agrees almost exactly with the value for the 1-2 peak predicted by Kahn.[9] The maximum at approximately 690 cm^{-1} should belong to the 1 - 3 transition. The position of this latter maximum gives, with the aid of Kahn's formula, the spin-orbit splitting in the origin $E_1 - E_3 = 0.05$ ev. From this formula we also get the position of the maximum arising from the 2 - 3 transition, viz., approximately 290 cm^{-1}. With the present apparatus, it is not possible to examine this region.

The possible maximum at about 530 cm^{-1} seen in Fig. 2 should not, according to this interpretation, be due to the same reason as an apparently analogous peak in Ge but can instead be caused by, for example, an impurity level (see below).

The ratio of 0.05 ev and the corresponding value, 0.28 ev, for germanium agrees fairly well with the quotient of the tripiet splittings in the lowest states of the Si and Ge atoms, respectively.

Another interpretation, considerably less probable, is that the maximum at 690 cm^{-1} should be due to the 2 - 3 transition. In this case, the 1 - 3 maximum would be situated at about 1000 cm^{-1} and the spin-orbit splitting would have the improbably high value of 0.10 ev. No band absorption due to free carriers has been reported

in this region in spite of the numerous investigations on p-type silicon, nor have we found in the present investigation any absorption band which can be attributed to free hole absorption.

Indeed we have observed at about 1000 cm^{-1} bands which, however, show the typical appearance of impurity level absorption and which agree with the known levels of indium.[11] This observation, which will be reported in detail later, shows that, with the method used by us, it is possible to detect absorption due to impurity states at room temperature.

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COEFFICIENT OF THERMAL EXPANSION OF LIQUID He^3 NEAR 0°K*

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(Received October 8, 1958)

Recent calculations by Brueckner and Gammel[1] on the properties of liquid He^3 near 0°K have determined the specific heat and also its variation with density. We wish to emphasize the relationship between this result and the coefficient of thermal expansion. The predicted value of the latter may be of interest in the interpretation of the recent measurements by Lee, Reppy, and Fairbank[2] of the thermal coefficient.

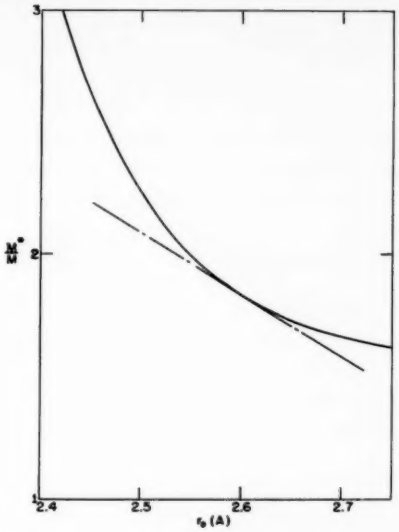

FIG. 1. Variation of M^*/M with density parameter r_0, as defined in Eq. (6). The dashed curve gives the tangent at $r_0 = 2.60$ A.

The specific heat at constant volume is given by Brueckner and Gammel in the form

$$C = (M^*/M)\, C_F, \tag{1}$$

where M^* is the effective mass for particle motion near the Fermi surface and C_F is the specific heat of the ideal Fermi gas,

$$C_F = N\left(\frac{\pi}{3}\right)^{2/3} \frac{k^2 M}{\hbar^2} \left(\frac{V}{N}\right)^{2/3} T. \tag{2}$$

The ratio M^*/M in Eq. (1) determines the shift in level density from that of the ideal Fermi gas. Figure 1 gives M^*/M as a function of density. From this result, it is possible to determine the derivative of M^*/M with respect to the volume in the vicinity of the equilibrium volume at zero pressure.

The result is

$$\frac{M^*}{M} = 1.84 + 2.25\, \frac{(V_0 - V)}{V_0}, \tag{3}$$

where V_0 is the volume at zero pressure. In the calculations of Brueckner and Gammel, this is

$$V_0 = (4/3)\, \pi N r_0^3, \tag{4}$$

with $r_0 = 2.60 \times 10^{-8}$ cm.

The relation between C and α, the coefficient of expansion, follows from the Maxwell equation

$$(\partial S/\partial P)_T = (\partial V/\partial T)_P = -\alpha V. \quad (5)$$

This definition of α is at constant pressure, which at equilibrium is at $P = 0$. The entropy, using Eq. (1), is given by the equation

$$S = \int_0^T \frac{C(T')}{T'}\, dT' = NM^* \left(\frac{\pi}{3}\right)^{2/3} \frac{k^2}{\hbar^2} \left(\frac{V}{N}\right)^{2/3} T, \quad (6)$$

where we have neglected any possible variation of M^* with temperature, which would give corrections of order T^2 and higher.

At low pressures and low temperature, the pressure can be written

$$P = \beta \frac{V_0 - V}{V_0}. \quad (7)$$

The parameter β as determined in the calculations of Brueckner and Gammel is 18.9 atmospheres; the experimental value obtained from the velocity of sound[3,4] is 27.5 atmospheres which we adopt as more accurate. Using Eqs. (5), (6), and (7), we thus obtain the result at $V = V_0$

$$\alpha = T\left(\frac{\pi}{3}\right)^{2/3} \frac{N^{1/3}}{\beta} \left[\frac{\partial}{\partial V}\left(M^* V^{2/3}\right)\right]_{V=V_0}. \quad (8)$$

Using Eq. (3), we find

$$\alpha = T\left(\frac{\pi}{3}\right)^{2/3} \frac{k^2 M}{\hbar^2} \frac{2}{3\beta} \left[1.84 - \frac{3}{2}(2.25)\right]. \quad (9)$$

The second term in the square bracket in Eq. (9) arises from the density variation of the effective mass. If it is neglected, then the coefficient of expansion is positive and the liquid would contract when cooled near 0°K. The density variation of the effective mass is sufficiently rapid, however, to change the sign of α. Inserting the numerical values of β and V_0, we find

$$\alpha = -0.076\ T\ (\text{deg K})^{-1}, \quad (10)$$

where T is measured in degrees Kelvin.

The result of Eq. (10) can be expected to hold only very close to 0°K, where the specific heat follows the simple linear law of Eq. (1) and Eq. (2). This probably limits the validity of Eq. (10) to temperatures below 0.2°K. At $T = 0.2$°K, we find that α is -0.0152 (deg K)$^{-1}$ corresponding to a decrease in volume from 0°K of 0.15%, i.e., the liquid contracts as it is heated.

This anomaly in the coefficient of expansion can be attributed to the dominant effects of the Fermi statistics at very low temperatures. When the liquid is cooled, the tendency of the He^3 atoms to form a state of momentum order, the degenerate Fermi gas, is inhibited by the effects of the strong forces. The repulsion is enhanced by the action of the exclusion principle in the degenerate state, while the attraction becomes less effective in the presence of high zero-point energy and short wavelengths. These two effects are less pronounced if the liquid density is decreased; consequently the liquid expands to "make room" for the atoms as they drop down into the state of momentum order.

*Supported in part by grants from the National Science Foundation.

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ALIGNMENT OF METASTABLE HELIUM ATOMS BY UNPOLARIZED RESONANCE RADIATION*

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(Received October 8, 1958)

This is a preliminary report of experiments in which helium atoms in the $^3S_1$ metastable state have been aligned by the action of unpolarized optical resonance radiation. The techniques and theory of this effect exhibit many similarities to "optical pumping" experiments,[1] but there are important dissimilarities. The alignment has been detected by resonance methods; the observed line widths are of the order of a milligauss with a raw signal-to-noise ratio of ~100:1. Our simplest experimental arrangement will be described, followed by an outline of the theory.

Pure helium at a pressure of 2 mm Hg is contained in a 12-in. long, 1½-in. diameter, cylindrical, Pyrex tube fitted with an aluminum disk electrode at each end. Within this tube helium atoms are continuously excited to the $^3S_1$ metastable state by a discharge maintained with a few milliamperes of either dc or rf current. Light from a bright helium lamp[2] is passed

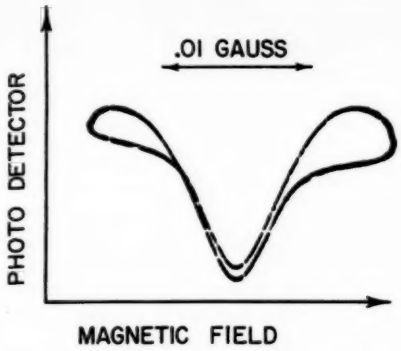

FIG. 1. CRO trace of the metastable helium resonance at 1.3 Mc/sec as seen in the earth's magnetic field. The field was modulated at 60 cycles/sec. The amplifier bandwidth was ~5 kc/sec. The signal-to-noise ratio was ~10^2 in the original photograph. The line width is due almost entirely to the inhomogeneity of the earth's magnetic field in the laboratory.

through the discharge tube along the direction of the earth's magnetic field. This light is received by a Kodak PbS "Ektron" detector, the output of which is amplified and displayed on a CRO. A radio-frequency magnetic field is produced at right angles to the earth's field by means of several turns of wire wrapped arount the discharge tube and driven by a General Radio Unit Oscillator. Helmholtz coils are used to modulate the earth's field synchronously with the horizontal trace of the CRO.

The resonance shown in Fig. 1 was obtained with this apparatus. The resonance frequency was 1.5 Mc/sec corresponding to a "g-value" of 2.8 Mc/gauss. A signal of the same sense and roughly the same magnitude is observed when the optical resonance radiation is passed through the discharge tube at 90° with respect to the earth's field.

The features of the helium energy level diagram relevant to this experiment are shown in Fig. 2. By "resonance radiation" is meant that light emitted in transitions from the set of closely spaced (1S2P) ^3P levels to the $^3S_1$ metastable levels. The light originating from the $^3P_2$, $^3P_1$, and $^3P_0$ states will be denoted as D_2, D_1, and D_0 radiation, respectively. In a typical helium arc the D_2 and D_1 radiation are totally unresolved, but completely separate from the D_0 radiation. Thus the D_2 and D_1 radiations have equal intensity and, from consideration of the statistical weights, should be greater than the D_0 light. Let it then be assumed that the intensities of the D_2, D_1, and D_0 radiations are in the ratio of 1:1:K.

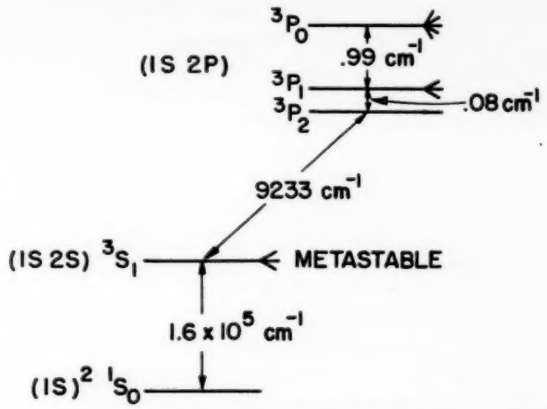

FIG. 2. Relevant features of the helium energy level diagram (not to scale).

A beam of this radiation is now projected through the dilute sea of metastable helium atoms which are continuously produced in the discharge tube. Let the rates at which the m=1, m=0, and m=-1 states of $^3S_1$ absorb this resonance radiation be denoted by R_1, R_0, and R_{-1}, respectively. If the light is projected collinearly with the magnetic field, it can be shown[3] that $R_1:R_0:R_{-1}=5+K:6:5+K$. Since the ^3P states are thoroughly mixed by collisions in the discharge tube, they will reradiate back to the $^3S_1$ levels without preference. (In the absence of this mixing, as in a free space experiment, the following conclusions are affected by a few percent.) Let the populations of the m=1, m=0, m=-1 levels of $^3S_1$ be denoted by n_1, n_0, and n_{-1}, respectively. If relaxation effects are neglected, it follows that the steady state populations will be 6:5+K:6. Thus there is an alignment in the $^3S_1$ states provided K differs from unity.

This alignment can be diminished by an rf magnetic field at the resonance frequency determined by the $^3S_1$ splitting. An inspection of the values of R_1, R_0, and R_{-1} reveals that more resonance radiation would now be absorbed by the metastable atoms, which accounts for the sign of the resonance signal shown in Fig. 1.

If the resonance radiation is projected at right angles to the magnetic field, $R_1:R_0:R_{-1}=(11+K):(10+2K):(11+K)$. This leads, in the absence of relaxation effects, to an alignment specified by $n_1:n_0:n_{-1}=(10+2K):(11+K):(10+2K)$. The resonant rf field will diminish this alignment which again produces an increased absorption of the reson-

ance radiation. Thus the same sense of signal is observed as when the light was collinear with the magnetic field.

It is easily shown that no alignment can be produced when the light beam makes an angle of $\cos^{-1}(\frac{1}{3})^{1/2} \cong 55°$ with the field axis. This follows from a consideration of the sum rules[3] and is independent of the values for the transition matrix elements. This prediction has been verified with the described experimental arrangement.

The effect of relaxation processes is to decrease the alignment and hence the resonance signal. We believe the relaxation time is of the order of one millisecond for our systems and is due to a quenching of the metastable states by impurities, including electrons, that are present in the discharge. Thus the resonance should exhibit a width of $\sim 10^{-4}$ gauss in a perfectly homogeneous magnetic field. We have observed line widths of $\sim 6 \times 10^{-3}$ gauss, which is consistent with the measured inhomogenieties of the earth's magnetic field presented by the Randall Laboratories. We have moved the apparatus into an adjacent parking area and observed line widths of $\sim 10^{-3}$ gauss, which appears consistent with the inhomogenieties of that area.

The helium pressure in the discharge tube may be varied over a range of about 10^2. The design of the discharge tube is not critical to the experiment. We have used dc excitation and rf excitation applied either to the electrodes or externally to the glass envelope.

The analysis outlined above assumed a dilute sea of metastable atoms—specifically, one in which the optical mean free path for the resonance radiation was large compared to the dimensions of the discharge tube. The dense-sea case is very complicated and gives rise to novel experimental effects such as a total inversion of the resonance line. The discussion of these effects is elaborate and will be presented in later papers together with a detailed discussion of the simple effects and their applications.

*This work was supported in part by the Faculty Research Fund of the Horace H. Rackham School of Graduate Studies of The University of Michigan, and the U. S. Atomic Energy Commission.

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FAR INFRARED ENERGY GAP MEASUREMENTS IN BULK SUPERCONDUCTORS*

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(Received October 13, 1958)

Several measurements[1] of the surface resistance of superconductors have been made at microwave frequencies with photon energies comparable with kT_c. In these experiments the surface resistance has generally approached zero for temperatures much less than T_c, showing that the photons used were not energetic enough to excite electrons across the absolute zero energy gap of $\sim 3.5\ kT_c$.[2,3] Using far infrared techniques, we have measured the onset of absorption in superconducting tin and lead for $T \ll T_c$ over the quantum energy ranges of (1 to 13)kT_c for lead and (2 to 18)kT_c for tin, corresponding to a wavelength range of ~0.15 to 2 mm. For both lead and tin these energy ranges span the transition from essentially lossless reflection to absorption indistinguishable from that of the metal in the normal state.

The output of a far infrared grating monochromator was conveyed through a brass light pipe to a cast nonresonant superconducting cavity immersed in liquid helium at 1.55°K. After making many reflections to build up the metallic absorption to a measurable amount, the radiation was absorbed in a carbon resistance bolometer mounted on a wall of the cavity. The power (P_S) reaching the bolometer when the cavity was superconducting was compared to that (P_N) when it was held normal by a magnetic field. The percentage change of the bolometer signal $[(P_S - P_N)/P_N]$ for tin and lead is plotted against photon energy in units of kT_c in Fig. 1.

For quantum energies below the energy gap these curves show the difference between the nearly perfect reflection of the superconducting state and the frequency-dependent reflection of the normal state. At quantum energies high enough to excite electrons across the energy

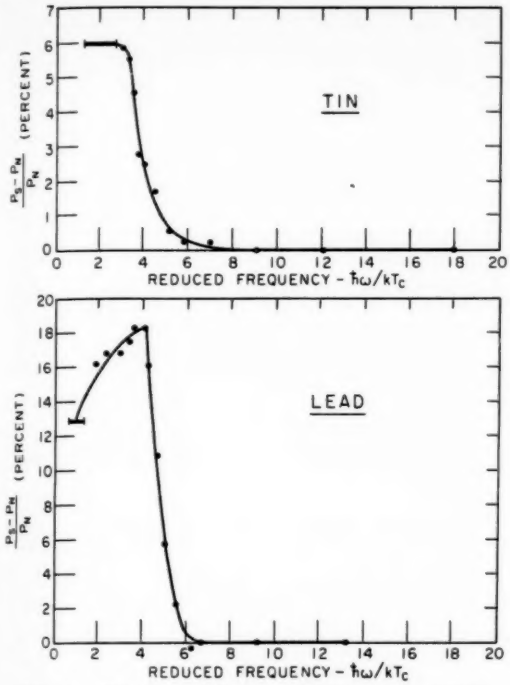

FIG. 1. Observed differences between the power reaching the bolometer in the normal and superconducting states as a function of frequency in lead and tin. Note that the sudden onset of absorption in the superconductor that marks the energy gap occurs at a slightly higher value of the reduced frequency in lead than in tin.

gap there is a sudden onset of absorption in the superconductor, and as the absorption approaches that in the normal state, $[(P_S - P_N)/P_N]$ falls rapidly to zero. The observed onset of absorption in the superconductor is qualitatively similar to that predicted by the BCS theory,[4] but is about twice as fast. This rapid approach to the normal-state value of the absorption makes it clear why earlier measurements[5] with room-temperature black-body radiation peaked at 0.014 mm revealed no change in absorption between the superconducting and normal states.

The scale of the ordinates is governed by the value of the normal-state reflectivity and by the amount of flux trapping. Trapped flux keeps $\frac{1}{4}$ to $\frac{1}{2}$ of the cavity in the normal state at all times after the magnet is first turned on. This normal fraction remains constant throughout a run. The presence of trapped flux in the cavity indicates that large amounts of the surface are subjected to fields of the order of H_c. Thus the sharpness of the absorption edge indicates that the energy gap width is not a sensitive function of such fields. If the law of corresponding states between superconductors were obeyed exactly, the tin and lead curves would be identical except for the above mentioned differences in flux trapping and normal state reflectivity.

The frequencies of interest for the determination of the energy gap in lead lie entirely within the range available from the monochromator. For tin, the required frequencies are reduced by the ratio of the transition temperatures (3.7/7.2 ~ 0.5), and the points at $3.0kT_c$ and $3.25kT_c$ were measured only with a sacrifice in radiation purity. Any admixture of higher-order radiation depresses these points, and this effect probably accounts for the difference between the two curves near E_g. The lowest frequency point on each curve was measured using broad-band radiation from a zero-order filter grating with the approximate spread of frequencies indicated by the horizontal bars. The frequency band width (~10%) used on all other points was not large enough to broaden the absorption edge appreciably. The accuracy in the vertical direction is limited by random noise (each point is an average of 8 to 10 readings) and by radiation impurity.

If one assumes that the sudden onset of absorption occurs at the gap energy E_g, then the widths of the energy gap in lead and tin at 1.55°K may be deduced to be $(4.1 \pm 0.1)kT_c$ and $(3.3 \pm 0.3)kT_c$, respectively. Extrapolating these results to 0°K using the BCS temperature dependence, and noting that 1.55°K is $0.22T_c$ for lead and $0.42T_c$ for tin, we find the widths of the energy gap at absolute zero, $E_g(0)$, to be $(4.1 \pm 0.2)kT_c$ for lead and $(3.4 \pm 0.3)kT_c$ for tin. These results may be compared with the BCS value of $3.5kT_c$, which should hold for all superconductors. The observed departure from such a simple law of corresponding states, with $E_g(0)/kT_c$ increasing slowly as T_c/θ_D increases, is of the sort pointed out by Goodman[6] on the basis of nonspectroscopic data.

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*This research was supported in part by the Office of Naval Research and the National Science Foundation.

†National Science Foundation Predoctoral Fellow.

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FERROELECTRICITY IN $NaNO_2$

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It is well known that sodium nitrite ($NaNO_2$) has the orthorhombic structure (C_{2v}^{20}) at room temperature[1] and this structure changes to the more highly symmetric one (D_{2h}^{25}) at about 160°C.[2,3] The piezoelectric effect observed in the low-temperature form disappears at this temperature.[2] Although the structural investigations of $NaNO_2$ crystal have been carried out well by several researchers,[4,5] other properties, in particular the dielectric properties, have scarcely been noticed.

We have studied the dielectric properties of this crystal and found that it is ferroelectric along the b axis below the 160°C transition point. Crystals were prepared by the fusion method, where $NaNO_2$ powder had been dried in advance in 10^{-3} mm Hg vacuum at about 120°C during 48 hr, since this substance is considerably deliquescent. Crystals were transparent, slightly yellowish, and easily cleaved along the (101) plane. Figure 1 shows the dielectric constant vs temperature relation measured at the frequency of 100 kc/sec, using air-drying silver paste electrodes. The dielectric constant in the direction of b axis, ϵ_{010} (dimension of specimen: thickness 0.0835 cm, area 0.537 cm²), is 7.4 at room temperature and reaches 780 at 164°C on heating and 1100 at 162°C on cooling, respectively, where the rate of temperature change was about 0.5°C/min. Above this transition temperature, the Curie-Weiss law holds with the Curie-Weiss temperature of 433°K and the Curie constant of 5.0×10^3°K. The dielectric constant perpendicular to the b axis, ϵ_{101} (dimension of specimen: thickness 0.114 cm, area 1.032 cm²), is 6.4 at

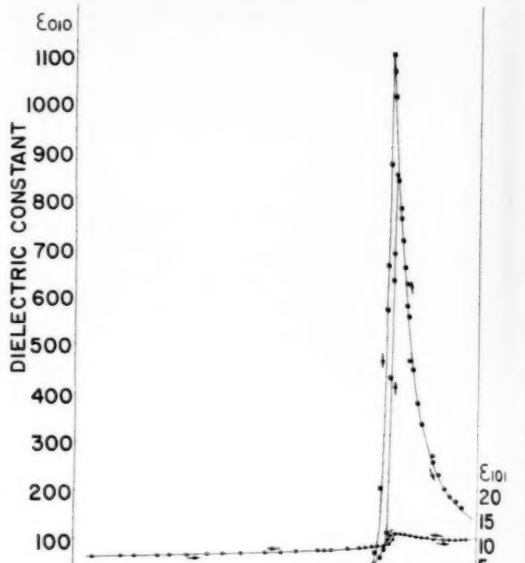

FIG. 1. Dielectric constant ϵ_{010} and ϵ_{101} (100 kc/sec) vs temperature. Squares: ϵ_{010}; circles: ϵ_{101}.

room temperature and remains low at high temperatures, though a small anomaly was observed at the transition.

The pyroelectricity was clearly observed along the b axis below the transition temperature and furthermore its polarity could be easily reversed by applying a dc field to the crystal. We obtained about 7 μcoul/cm² as the value of spontaneous polarization at room temperature from the pyroelectric measurement, using a sample which had been cooled to room temperature from 180°C applying the dc field of 1.4 kv/cm. The typical D-E hysteresis loop was observed below the transition temperature in the direction of b axis at 50 cps. An example is shown in Fig. 2, where the coercive field and the spontaneous polarization are seen to be 2.3 kv/cm and 6.4 μcoul/cm² at 143°C, respectively. This value of spontaneous polarization nearly coincides with that obtained from the measurement of pyroelectricity. The coercive field derived from the hysteresis loop varied with temperature and strongly depended on the applied field strength. At room temperature, the loop did not open out even for the field of 25 kv/cm, because of its large coercive field. A double hysteresis loop could be observed just above the transition tem-

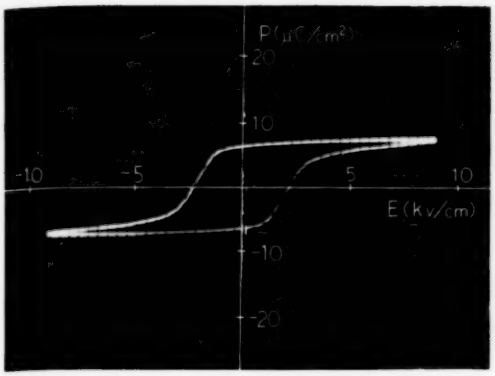

FIG. 2. Hysteresis loop (50 cps) in the direction of b axis at 143°C.

perature.

Judging from these experimental results, it is clear that $NaNO_2$ is ferroelectric along the b axis below the 160°C transition point, and the transition seems to be of the first kind, but even then the jump at the transition would be very small. The ferroelectricity is considered to come from the favorable arrangement of groups in the crystal. The ordered arrangement of these groups causing the crystal to be polar in the low-temperature form seems to become disordered at the transition temperature.

Details of our studies will be reported shortly.

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SUSCEPTIBILITY OF THE ISING MODEL OF AN ANTIFERROMAGNET

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The susceptibility versus temperature curve for an antiferromagnet is usually characterized by the appearance of a maximum and the precise relationship of this maximum to the associated anomaly in the specific heat is of great interest. The elucidation of this problem using even a comparatively simple model of an antiferromagnet, such as the Ising model, is difficult and the behavior in the critical region has long been a subject of speculation. The work summarized below, however, leads to the conclusion that the susceptibility, χ, versus temperature, T, plot for an antiferromagnet should have a vertical tangent (χ increasing with T) at the same temperature at which the specific heat anomaly occurs. Since χ must vanish as T becomes infinite, this form of the singularity implies that the observed maximum in χ occurs above the true critical temperature.

These conclusions are based on a study of the susceptibility of Ising models. No exact closed expressions for the susceptibility of two- or three-dimensional Ising models have yet been given but series have been published by various authors.[1-3] The best method of obtaining these seems to be that employed by Oguchi[2] and developed by Domb and Sykes.[3] Extrapolation of these series has been attempted, most recently by Park,[4] but it has proved difficult to draw firm conclusions in the critical region. We have been able to rewrite the general expansion in such a way that the major part of the susceptibility is known if the configurational energy of the lattice is known. The remaining part has a much simpler configurational interpretation than the original expansion. In terms of the antiferromagnetic "high-temperature" variable $v = \tanh |J|/kt$, where J is the interaction energy between parallel spins, we find that the susceptibility per spin may be written

$$\chi(v) = \frac{m^2}{kT}\left[1+(q-1)v\right]^{-2}\left\{1+(q-2)v+v^2 - 2v\left|\frac{U(v)}{J}\right| + 8\sum_{\gamma}(-1)^{\gamma}d_{\gamma}v^{\gamma}\right\}, \qquad \text{(A)}$$

where m is the magnetic moment per spin, q is the coordination number, and $U(v)$ is the configurational energy per spin. The positive coefficients d_γ enumerate a well defined class of closed graphs on the lattice with at most two odd vertices; d_0 to d_4 vanish identically on all lattices and d_5 and d_6 vanish on most loose-packed lattices.

By using this result, the series expansion for the susceptibility of the plane triangular lattice has been extended up to the term in v^{12}, that of the plane quadratic lattice up to the term in v^{14}, and that of the honeycomb lattice up to the term in v^{24}. The general result holds equally for three-dimensional lattices and the corresponding series can also be considerably extended.

Numerical evaluation of the function defined by $\Sigma(-1)^{\gamma}d_{\gamma}v^{\gamma}$ shows that it contributes only a few

percent or less to the value of χ at temperatures in the region of the critical point and above. (For lattices such as the plane triangular lattice which have no critical point, the relevant range is determined by the corresponding ferromagnetic Curie temperature.) Because of this the behavior of the susceptibility in the critical region is effectively determined by the energy alone. For plane lattices this is known exactly from the work of Onsager[5] and others,[6] and the expressions are valid at all temperatures. Consequently we may conclude that for loose-packed lattices, χ behaves in the critical region as $a+b(T-T_C)\log|T-T_C|$ where a and b are constants and T_C is the critical temperature. The energy is not known in closed form for three-dimensional lattices but the calculations of Wakefield[7] and of Domb and Sykes[8] indicate that the specific heat of a loose-packed lattice becomes infinite at T_C which again shows that the susceptibility should have a vertical tangent at the same temperature.

For close-packed lattices such as the plane triangular and the face-centered cubic, there is no specific heat singularity and the general result indicates that the susceptibility should not exhibit any singularity in the neighborhood of the corresponding ferromagnetic critical temperature.

The conclusion for plane lattices is borne out by an exact solution in the presence of a magnetic field of a particular class of two-dimensional Ising model in which normal Ising spins (with magnetic moments) are coupled together via "nonmagnetic spins" so yielding a type of "superexchange" interaction.

The generalization of the result (A) for spin $s>\frac{1}{2}$ has not been obtained. General arguments, however, indicate that similar physical conclusions should still be valid.

More detailed accounts of these investigations will be published in due course.

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CONDUCTION ELECTRON-MAGNETIC ION INTERACTION IN RARE EARTHS*

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(Received September 24, 1958)

The $4f$ electrons in the rare earths, which are responsible for the magnetic moments of the rare earth elements, are highly localized and presumably do not form a band; hence the direct interaction between $4f$ electrons on different atoms is expected to be fairly weak. The ion-ion interaction is presumably an indirect interaction via the conduction electrons in the manner suggested by Zener.[1] This conduction electron-magnetic ion interaction should also be manifested by the electrical resistivity.[2-4] It now appears that this interaction depends on the spin of the $4f$ shell and is involved in the electric and magnetic properties of the rare earth metals and in the superconductivity of alloys of the rare earths.

Electrical resistivity measurements on many of the rare earth metals have now been reported.[5-7] Figure 1 shows the electrical resistivity of Gd as a function of temperature. This is rather typical of the hexagonal close-packed rare earths Gd, Tb, Dy, Ho, Er, Tm, and Lu. The other rare earths do not show regions where the electrical resistivity varies linearly with temperature, so the present discussion is not pertinent for these elements. Following the proposal of Kasuya,[4] the electrical resistivity of a rare earth in the paramagnetic region might be ex-

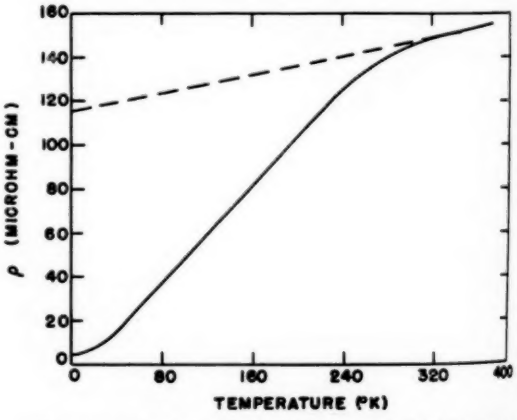

FIG. 1. The electrical resistivity of gadolinium as a function of temperature.

Table I. Resistivity values in microhm-cm.

Element	ρ_{res}	ρ_{corr}	ρ_{magn}
Gd	3.5	1.5	115.5
Tb	3.5	2.5	88.5
Dy	18.0	5.0	74.5
Ho	6.0	5.0	39.0
Er	8.5	4.5	25.4
Tm	28.7	4.4	10.2
Lu	12.0	4.9	1.6

pressed in the form $\rho=\rho_{th}+\rho_{res}+\rho_{magn}$ where ρ_{th} is the resistivity due to phonon scattering of electrons, ρ_{res} is the residual resistivity, and ρ_{magn} is the term arising from the magnetic disorder scattering of electrons.[8] ρ_{magn} is determined by extrapolating the resistivity in the paramagnetic region to 0°K, and subtracting the residual resistivity and a correction ρ_{corr} using the Grüneisen temperature dependence for the usual ideal resistivity. These quantities are shown in Table I. In Fig. 2 we have plotted ρ_{magn} as a function of $S(S+1)$. This indicates that an exchange interaction between the conduction electrons and the spin of the magnetic ions is one of the factors responsible for the scattering of conduction electrons. The ρ_{magn} term does not depend on J or gJ as one might expect.

Néel has previously noted that paramagnetic Curie temperatures of the rare earths follow a relation which can be deduced by assuming an interaction between spins only.[9] It is reasonable to assume that the exchange interaction between conduction electrons and the spin of magnetic

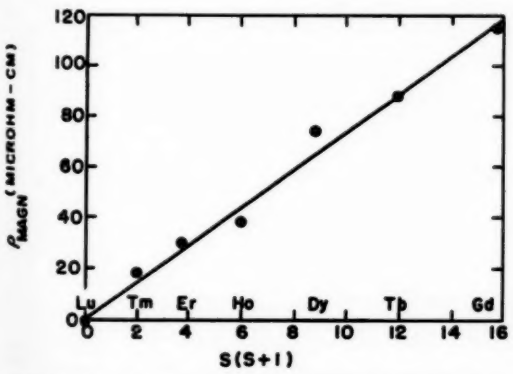

FIG. 2. The magnetic disorder contribution to the electrical resistivity as a function of $S(S+1)$ for the hexagonal close-packed rare earths Gd to Lu.

ions could result in an effective interaction between the spins of the magnetic ions.[1]

Recently it has been shown with alloys of rare earths and lanthanum that the depression of superconductivity is correlated with the spin of the solute atoms.[10] We believe that the conduction electron-magnetic ion interaction, which we have just discussed, is responsible for this shift. According to the BCS theory of superconductivity,[11] the superconducting transition temperature may be expressed as

$$T_c=(1.14/k)\langle\hbar\omega\rangle\exp[-1/N(0)V],$$

where $\langle\hbar\omega\rangle$ is the average energy of phonons interacting with electrons, $N(0)$ is the density in energy of electrons of one spin, and V is the average strength of the net electron-electron interaction (all quantities being defined at the Fermi surface). V may be expressed as

$$V=\left\langle\frac{2|M_K|^2}{\hbar\omega_K}-\frac{4\pi e^2}{\kappa^2}\right\rangle_{Av},$$

where $4\pi e^2/\kappa^2$ is the screened Coulomb interaction and $2|M_K|^2/\hbar\omega_K$ is the phonon induced effective electron-electron interaction. The criterion for superconductivity is that $V>0$. Since we will be concerned with solid solutions with the order of 1% solute atoms, we assume that $\langle\hbar\omega\rangle$ does not vary much. We also assume that $N(0)$ and $4\pi e^2/\kappa^2$ also are not changed much since the total number of conduction electrons per atom is unchanged and the volume change is very slight. These assumptions seem justified by experiments performed on similar alloys with solute atoms of zero spin.[12] Our proposal is that there is an additional effective electron-electron interaction which results from the conduction electron-magnetic ion exchange. V should then be expected to be a function of spin (for these alloys) as well as solute concentrations. In Figs. 3 and 4 we have shown the data of Matthias, Suhl, and Corenzwit[10] fitted to an empirical equation $N(0)V=[N(0)V]_{La}-a\delta[S(S+1)]^2$, where $[N(0)V]_{La}$ is the value for pure face-centered cubic lanthanum, a is the percent magnetic rare earth, S is the spin quantum number for the $4f$ electrons of the solute atoms, and δ is a proportionality constant independent of the solute atoms. The shift in transition temperature due to a volume change would be negligible for these alloys. The largest shift we would expect for 1% magnetic ion would be 0.1 degree.[12]

It is also possible that the shift may be explain-

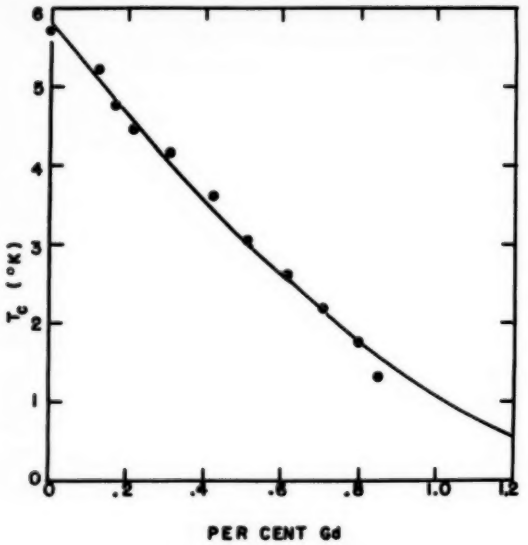

FIG. 3. Superconducting transition temperatures of La-Gd alloys. The points are the data of Matthias, Suhl, and Corenzwit and the solid curve is theoretical.

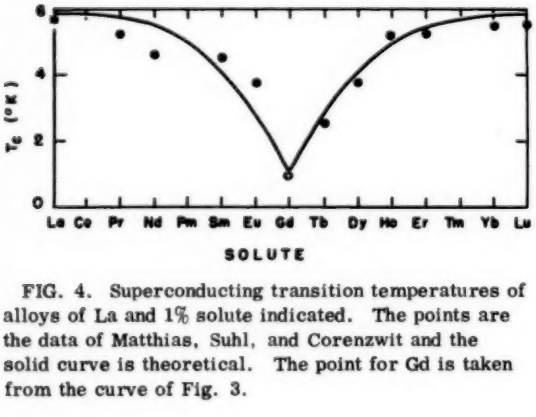

FIG. 4. Superconducting transition temperatures of alloys of La and 1% solute indicated. The points are the data of Matthias, Suhl, and Corenzwit and the solid curve is theoretical. The point for Gd is taken from the curve of Fig. 3.

ed in a manner suggested by Pippard[13] for impurity scattering of electrons.

In his model $|M_\kappa|^2$ depends on κl in the same manner as the phonon mean free path (κ is the longitudinal wave number and l is the electronic mean free path). A decrease in l results in a decrease in $|M_\kappa|^2$ and hence in T_c.

The ferromagnetic impurity effect has also been treated recently by Herring[14] and by Matthias and Suhl.[15]

*Contribution No. 673. Work was performed in the Ames Laboratory of the U. S. Atomic Energy Commission.

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TEST OF TIME-REVERSAL INVARIANCE OF THE BETA INTERACTION IN THE DECAY OF FREE POLARIZED NEUTRONS*

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Jackson, Treiman, and Wyld[1] have pointed out that the electron momentum, $\vec{p}_e$, the antineutrino momentum, $\vec{p}_{\bar{\nu}}$, and the polarization of the nucleus, $\langle \vec{J} \rangle / J$, may exhibit, in the decay of oriented nuclei, a correlation of the form

$$1 + \mathfrak{D}(\langle \vec{J} \rangle / J) \cdot (\vec{p}_e / E_e) \times (\vec{p}_{\bar{\nu}} / E_{\bar{\nu}})$$

if, and only if, the interaction responsible for beta decay is not invariant under time reversal.

Other three-vector-correlation experiments that would test time-reversal invariance have been suggested[1-3] and in part already carried out.[4] However, a search for the correlation mentioned above in the decay of free polarized neutrons appears to be of considerable interest since this process affords a particularly sensitive test, and its interpretation requires no knowledge of nuclear matrix elements and virtually no Coulomb corrections. One has simply

$$\mathfrak{D} = -\frac{2\,\mathrm{Im}(C_V C_A^* + C_V' C_A'^*)}{|C_V|^2 + |C_V'|^2 + 3(|C_A|^2 + |C_A'|^2)}, \quad (1)$$

assuming, as is consistent with recent experiments,[5] that no couplings of S and T type occur in beta decay.

In continuation of our work on the decay of polarized neutrons,[6] we have carried out an experiment to set an upper limit for $\mathfrak{D}$ in this process. This experiment was performed by making a slight modification in the apparatus used to measure the correlation[5] between the spin of the neutron and the direction of emission of the antineutrino. Its principle will be explained with reference to Figs. 1 (a) and 1 (b). Figure 1 (a) represents the "normal", Fig. 1 (b) the "time-reversed" situation. Consider the plane of the figure as the x-y plane, and a neutron polarized along the $+z$ axis decaying, at the point indicated by x, with $\vec{p}_e$ along $+x$ and $\vec{p}_{\bar{\nu}}$ along $+y$. Assuming these momenta to have about equal magnitudes, the slit system indicated in Fig. 1 (a) will transmit the recoil proton ($\vec{p}_p$) to the proton detector, thus enabling it to make a coincidence with the electron. In the "time-reversed" situation, $\vec{p}_e$ and $\vec{p}_{\bar{\nu}}$ will appear reversed, and the neutron spin $\vec{J}$ will point along $-z$, so that the slits and the two detectors will have to assume the positions indicated in Fig.

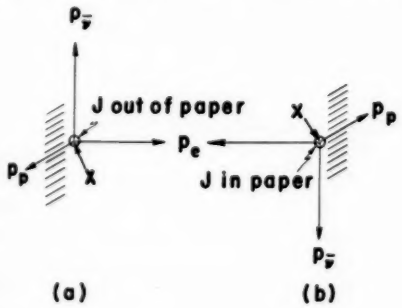

FIG. 1. Schematic drawing of arrangement for "time-reversed" measurements.

Table I. Results of measurements on W, the correlation between antineutrino momentum, $\vec{p}_{\bar{\nu}}$, electron momentum, $\vec{p}_e$, and neutron spin, $\langle\vec{J}\rangle$,

$$W = 1 + \mathfrak{D}\frac{\langle\vec{J}\rangle}{J}\cdot\frac{\vec{p}_e}{E_e}\times\frac{\vec{p}_{\bar{\nu}}}{E_{\bar{\nu}}}.$$

Beta energy	150 - 650 Mev
Observed asymmetry	-0.015 ± 0.017
Correction factor for angular spread of antineutrinos	2.2
Polarization of neutrons $\langle\vec{J}\rangle/J$	0.87
Mean p_e/E_e	0.8
$\mathfrak{D}$	-0.04 ± 0.07

1 (b) to give a coincidence. The coincidence rates per neutron should be the same with both arrangements if the process is invariant under time reversal. In practice, it will suffice to compare coincidence rates for neutrons polarized in the $+z$ and $-z$ directions, since the two arrangements indicated can be made to coincide by a rotation around the z axis, an operation which is still assumed to leave physical laws invariant.

To perform the actual experiment, the slit system shown in Fig. 1 of reference 5 was rotated by 90° along a line normal to the face of the beta counter indicated in that figure, so that recoils associated with antineutrinos emitted perpendicularly to the neutron spin were preferentially collimated onto the proton counter. All other features and the operating conditions were essentially unchanged with respect to those described. In consequence of a recent re-activation of the proton detector, the ratio of coincidences to background (at about 0.3 coincidences/min) had the rather favorable value of 3/1. The results of the measurements are indicated in Table I, where the errors given in the first line are purely statistical. The error in $\mathfrak{D}$ includes an uncertainty of ± 0.03 because of possible misalignment of the neutron spin which brings in a contribution from the correlation between the antineutrino momentum and the neutron spin. The preliminary result of a similar experiment[8] is in agreement with our result.

Expression (1) could yield a nonzero result if there was a phase difference, other than 0 or π, between any of the four constants. To express the experimental result in terms of a single

phase angle θ, we shall assume that $C_A = x\,C_V e^{i\theta}$, and $C_A' = C_A$, $C_V' = C_V$, with x and C_V real. The relations between primed and unprimed constants correspond to right-handed antineutrinos; the absence of phase differences between primed and unprimed members of either pair is strongly suggested by the high polarization of beta particles. Taking $x = +1.19 \pm 0.04$,[7] which is consistent with our most recent value ($\mathfrak{A} = -0.11 \pm 0.02$) for the correlation between electron momentum and neutron spin, we conclude that θ does not differ from π by more than $\pm 8°$. It is to be noted that the combination of our values of $\mathfrak{A}$ and $\mathfrak{B}$ already implies this difference is not more than 45°.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

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ANGULAR CORRELATION OF ALPHA PARTICLES FROM DECAY OF Li^{8*}

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The determination of the electron-antineutrino correlation in an allowed transition has for many years constituted a basic experimental approach to the solution of the problem of the nature of the beta-decay interaction. In the case where only the directions of the electron and antineutrino are observed, the correlation is given by

$$W(\theta_{\beta\bar{\nu}}) = 1 + \alpha(v/c)\cos\theta_{\beta\bar{\nu}} ,$$

where α has the values 1, 1/3, -1/3, -1 for vector, tensor, axial vector, and scalar interactions respectively. As a practical matter, the direction of the antineutrino emission is ordinarily deduced from coincident measurements of the vector momenta of the electron and the recoil nucleus. Even so, the experiment involves great difficulties because of the extremely low energy of the heavy recoil particle. Some years ago[1] it was pointed out that the decay of Li^8 provides a particularly attractive object of study in this regard, both because the decay energy is relatively high and because the kinematics of the alpha-particle decay of the daughter nucleus, Be^{8*}, permits extraction of the necessary information from observations on the relatively high-energy, easily detectable alpha particles.

Radioactive Li^8, which has a half-life of 0.84 sec, decays to the 2.9-Mev, $J^\pi = 2^+$, excited state of Be^8, which immediately breaks up into two alpha particles. The half-width of the Be^{8*} is about 1 Mev. In the Be^{8*} frame of reference the two alpha particles emerge with equal energies, and in opposite directions. In the laboratory frame, the effect of the recoil is seen as a deviation in angle between the alpha particle paths or as a difference in their energies, depending upon the direction of recoil. With a total beta transition energy of 13.6 Mev, the maximum deviation in angle is about 7 degrees and the maximum energy difference is about 20%. It is clear that determination of the vector momenta of the electron and the two alpha particles uniquely determines the vector momentum of the antineutrino, and with a sufficient number of cases, permits the desired distinction between the V, T, A, or S interactions.[1,2] On the other hand, the much simpler experiment in which only the distribution in angle between the alpha particles is observed also permits certain distinctions, though of a more restricted character. In effect, such a distribution is a direct measure of the distribution of recoil momenta perpendicular to the line of emission of the alpha particles. Qualitatively, it may be seen that if the electron and the antineutrino are emitted mainly parallel to one another (vector interaction), the mean square perpendicular recoil momentum will be large, while if they are most often antiparallel (scalar case), the recoil momentum will be small. The T and A interactions would be expected to

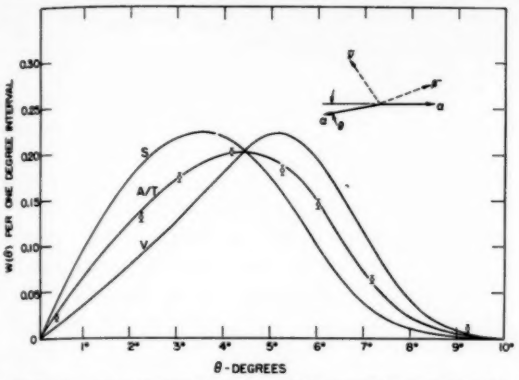

FIG. 1. The experimental points are the results of measurements on the distribution in θ, the angular deviation from 180°, in the laboratory frame of reference, between the directions of emission of the two alpha particles from the reactions Li^8 $(\beta^-,\bar{\nu})$ Be^{8*} $(\alpha)He^4$. The theoretical curves, corrected for finite experimental resolution, correspond to vector (V), tensor (T), axial vector (A), and scalar (S) interactions for beta decay assuming spin and parity 2^+ for both Li^8 and Be^{8*}. The theoretical curves for T and A are identical.

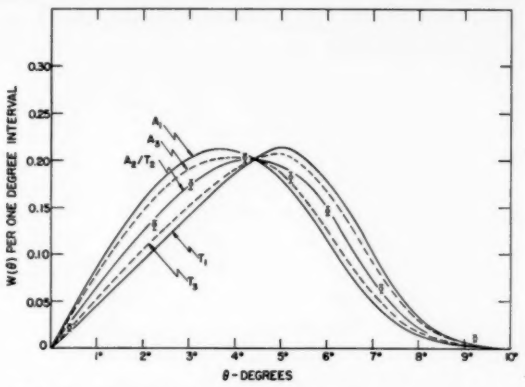

FIG. 2. The experimental points are the same as in Fig. 1. The theoretical curves apply to the following cases: A_1, A_2, A_3, axial vector interaction, T_1, T_2, T_3, tensor interaction both with $J^\pi(Li^8)=1^+$, 2^+, and 3^+, respectively.

produce intermediate cases. The actual theoretical distributions,[3] suitably modified for the conditions of the present experiment, are shown in Figs. 1 and 2, for various assumed values of the spin and parity of Li^8. Unfortunately, for the most probable assignment, $J^\pi=2^+$ for Li^8, the distributions for T and A are identical, so no distinction between these cases is possible in this experiment. On the other hand, any significant contribution of S or V interaction should be readily detectable. Early cloud chamber measurements[1] on the mean square perpendicular recoil momentum indicated a value twice that expected from the electron momentum alone. This result was consistent with an antineutrino momentum equal in average magnitude to that of the electron and with no correlation in the perpendicular components of the antineutrino and electron momenta. This is just what is now to be expected for both the T and A interactions. However, the detailed distributions were inconclusive because of the small number of alpha pairs photographed.

In the present experiment, the distribution in angle between the alpha particles was determined by counting techniques in order to obtain better statistics. The physical arrangement consisted of a Li^8 source, produced by deuteron bombardment of Li^7, placed midway between two aperture systems. The first was a pinhole collimator, 1.25 mm in diameter, subtending about 0.5 degrees at the source. For the other, a series of interchangeable annular apertures was provided, each subtending a hollow cone coaxial with the source and the pinhole. The cones so defined had radial widths of 0.5 degrees and mean opening angles ranging from 0.5 to 9 degrees. Coincidences between alpha particles passing through the two slit systems were recorded by means of two thin CsI scintillators, photomultipliers, and appropriate electronics. A crude magnetic analysis of the alpha particles passing through the pinhole collimator made it possible to reject all pairs of alpha particles except those whose energy corresponded approximately to the center of the 2.9-Mev state of Be^8. In all, some 6000 coincidences were recorded at eight different angles.

The results are shown by the open circles in Figs. 1 and 2, where the ordinates represent the number of coincidences observed per single alpha particle passing through the collimator and magnetic analyzer system, normalized to an angular interval of one degree. In comparing the observed coincidence distribution with the theoretical distributions, corrections were made for the finite angular resolution, for scattering in the source, and for the change in energy distribution due to energy loss in the source. These corrections have been folded into the theoretical curves of Figs. 1 and 2. The normalization of both the experimental points and the curves is absolute and independent. The agreement between the experimental points and the curve marked A/T in Fig. 1 leaves little doubt that, if $J^\pi(Li^8)=2^+$, the beta-decay interaction in this case is either A or T or a mixture of A and T. It would appear

unlikely from these results that there exists any S or V contribution as great as 10%: this result is of course to be expected from the fact that change in total isotopic spin is forbidden in Fermi interactions. Examination of Fig. 2 shows that the spin and parity of Li^8 cannot be 1^+ or 3^+, except in the unlikely contingency that A and T interactions contribute equally. We conclude that Li^8 has spin and parity 2^+, and that the beta-decay interaction is at least 90% Gamow-Teller. These conclusions are essential to the elucidation of the nature of the Li^8 beta-decay interaction as indicated by Barnes et al.[4]

* Supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

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NATURE OF THE Li^8 BETA-DECAY INTERACTION *

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It has been established in the preceding Letter[1] that the spin and parity of Li^8 are 2^+, and that the beta decay to the 2^+ excited state of Be^8 proceeds at least 90% of the time by a Gamow-Teller transition. With the spin sequence 2^+-2^+-0^+, a measurement of the recoil momentum from the electron and antineutrino, perpendicular to the direction of the alpha break up, distinguishes vector (V) and scalar (S) interactions from each other and from axial vector (A) and tensor (T) interactions, but does not allow a distinction between A and T.

The distinction between A and T can be made, however, by a determination of the distribution of recoil momenta along the alpha break-up direction.[2] This component will generally be larger when the electron and antineutrino are emitted preferentially in the same direction (V and T), and smaller in the opposite case (S and A). Theoretically, V and T give identical predictions for the spin sequence 2^+-2^+-0^+, and S and A are likewise indistinguishable.

In the present experiment, the Li^8 is produced by bombarding a thin layer of Li^7 on a 0.005-inch aluminum backing, with a deuteron beam pulsed at 60 pulses per second. During the periods when the beam is cut off, alpha particles following the Li^8 beta decay are magnetically analyzed and detected in a 0.002-inch thick CsI(Tl) crystal in coincidence with electrons travelling perpendicular to the alpha direction, or alternatively antiparallel to the alpha direction. The electrons are detected in collimated plastic scintillators so that pulse-height analysis can be carried out to select various ranges of electron energy. The measured alpha-particle momentum spectra for the two counter configurations, and for two different ranges of electron energy, are shown in Figs. 1 and 2. Corrections to the experimental data have been made for the He^+/He^{++} ratio and for the momentum loss of the alpha particles in escaping from the target.

To first order in p_β/p_α, the alpha-particle spectrum observed for the perpendicular counter configuration is just the spectrum in the rest system of the Be^{8*} slightly broadened symmetrically, for all interactions, by the antineutrino momentum. Also to first order, the momentum of an alpha particle in the antiparallel configur-

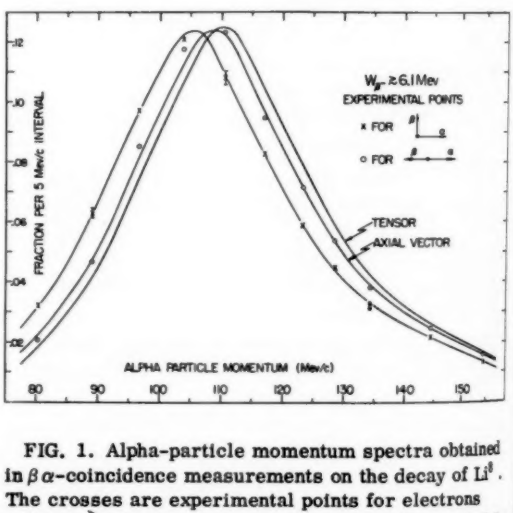

FIG. 1. Alpha-particle momentum spectra obtained in $\beta\alpha$-coincidence measurements on the decay of Li^8. The crosses are experimental points for electrons with $W_\beta \gtrsim 6.1$ Mev perpendicular to the observed alpha particle; the open circles are for electrons with $W \gtrsim 6.1$ Mev antiparallel to this alpha particle. Theoretical curves corrected for finite experimental resolution are shown for axial vector (or scalar) and tensor (or vector) interaction.

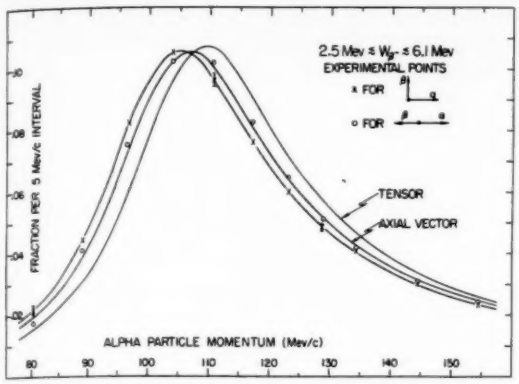

FIG. 2. Theoretical and experimental spectra as in Fig. 1 except for electrons with 2.5 Mev $\lesssim W_\beta \lesssim$ 6.1 Mev.

ation is increased by an amount

$$\Delta p_\alpha = \tfrac{1}{2}(p_\beta - p_{\bar\nu}\cos\theta_{\bar\nu\alpha}), \qquad (1)$$

where p_β and $p_{\bar\nu}$ are the absolute magnitudes of the electron and antineutrino momenta. The theoretically predicted angular correlation for antineutrino and alpha particle is, for $\cos\theta_{\beta\alpha}=-1$,

$$W(\theta_{\bar\nu\alpha}) = 1 \pm \frac{v}{c}\cos\theta_{\bar\nu\alpha}, \qquad (2)$$

where the plus sign holds for A or S and the minus sign for T or V.[2,3] When the antineutrino momenta are averaged over all angles, with the angular correlation given by Eq. (2), and average magnitudes inserted for the electron and antineutrino momenta, the spectrum of the observed alpha particles in the antiparallel configuration will be shifted upwards on the average by

$$\langle \Delta p_\alpha \rangle = \tfrac{1}{2}\langle p_\beta \rangle \pm \tfrac{1}{6}\langle p_{\bar\nu}\rangle, \qquad (3)$$

where the plus sign holds for T or V and the minus sign for A or S. Since the measurements described in the preceding Letter[1] set upper limits for S and V, we are here interested in distinguishing A from T. To calculate $\langle p_\beta \rangle$ and $\langle p_{\bar\nu}\rangle$ we have assumed that the spectra of electron and antineutrino momenta in coincidence with alpha particles of energy E_α are given by the usual Fermi expression with an end-point energy $W_0=(16.6-2E_\alpha)$ Mev.

The crosses in Fig. 1 give the experimental alpha-particle momentum spectrum for the perpendicular configuration, with a smooth curve drawn through the points. The other two curves in Fig. 1 are constructed from this smooth curve with the aid of Eq. (3), for electrons of total energy $W_\beta \gtrsim 6.1$ Mev. A small correction has been made for the beta counter pulse height resolution which was ~15% at 6.1 Mev. It is evident that the open circles, which give the measured spectrum for the antiparallel configuration, agree well with the axial vector prediction.

The curves in Fig. 2 are derived similarly for 2.5 Mev $\lesssim W_\beta \lesssim$ 6.1 Mev. In addition to the resolution correction already mentioned, corrections have been made for high-energy electrons scattered from the collimator, as determined experimentally, and for the "tail" of small pulses always present in the pulse-height spectrum of high-energy electrons. Again, it is evident that the observed shifted spectrum agrees well with the axial vector prediction. A qualitative comparison of the results in Fig. 2 with those in Fig. 1 is instructive. A relatively large shift is expected for either A or T in the antiparallel curve of Fig. 1 since the antiparallel electron has the greater share of the decay energy and the shift is thus not very sensitive to the orientation of the low-energy antineutrino. The measurements show that the experimental arrangement was capable of measuring such a relatively large shift. On the other hand, the small shift in Fig. 2 shows the partial cancellation to be expected for the axial vector interaction between the antiparallel momentum of the low-energy electron and the average parallel component of momentum of the high-energy antineutrino.

A more quantitative comparison of the experimental shifts with the theoretically expected shifts may be made by correcting all shifts for their dependence on p_α and then averaging. Table I displays the average shifts, corrected to

Table I. Experimental and theoretical momentum shifts (Δp_α in Mev/c) (Corrected to p_α = 100 Mev/c and averaged).

	Axial vector		Tensor	
W_β	Exp.	Theory	Exp.	Theory
$W_\beta \gtrsim 6.1$ Mev	3.55 ± 0.16	3.41	3.63 ± 0.17	5.18
2.5 Mev $\lesssim W_\beta \lesssim$ 6.1 Mev	1.50 ± 0.08	1.51	1.60 ± 0.10	4.23
$W_\beta \gtrsim 2.5$ Mev	2.61 ± 0.17	2.40	2.71 ± 0.17	4.67

$p_\alpha = 100$ Mev/c ($E_\alpha = 1.33$ Mev). The quoted errors are standard deviations of the mean values. It will be noted that the corrected experimental shifts differ only slightly for the two theoretical predictions.

The results show that the Li^8 beta decay proceeds by the axial vector interaction with an upper limit on the tensor interaction of approximately 10%. These results incidentally set a similar upper limit on the vector interaction but cannot exclude a large contribution from the scalar interaction. However, the preceding Letter sets an upper limit of 10% on this latter interaction. The two combined results establish the Li^8 decay as at least 90% Gamow-Teller and the Gamow-Teller portion as at least 90% axial vector.

Morita[3] has shown that the coefficient of the second term on the right-hand side of Eq. (2) is $-\frac{1}{3}$ for $J^\pi(Li^8) = 1^+$ and $+\frac{1}{7}$ for $J^\pi(Li^8) = 3^+$, so that the magnitude of the observed shifts exclude these possibilities as do the results of the preceding Letter. The conclusions of this and the preceding Letter are in accord with results reported on Li^8 by Lauterjung et al.[4] and on the nature of the Gamow-Teller interaction in He^6 as recently revised by Herrmannsfeldt et al.[5]

It is a pleasure to acknowledge our indebtedness to R. F. Christy, R. P. Feynman, M. Gell-Mann, and B. Stech for many helpful discussions.

*Supported in part by the joint program of the Office of Naval Research and the U.S. Atomic Energy Commission.

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CIRCULAR POLARIZATION OF EXTERNAL BREMSSTRAHLUNG PRODUCED BY BETA RAYS

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Beta particles longitudinally polarized on account of parity violation give rise to a circularly polarized external bremsstrahlung. This was shown by Goldhaber et al.[1] observing the transmission of the photons through magnetized iron. However, this method works only for photon energies above 600 kev. For smaller energies the polarization was measured by Galster and Schopper[2] using the forward scattering from magnetized iron. These results are in qualitative agreement with calculations[3,4] but a quantitative comparison has not yet been performed. The computations were done for monoenergetic electrons falling on a thin absorber whereas in all experiments the whole beta spectrum and thick absorbers were used. It will be shown here that even in this case a comparison with theory is possible as the diffusion of the electrons in the absorber has practically no influence on the polarization.

Cohen et al.[5] reported in one case a strong dependence of the circular polarization on the atomic number Z of the absorber. As this effect is rather unexpected we investigated it more carefully. The experimental setup was the same as that used in a former experiment.[2] The bremsstrahlung was produced by beta particles of $Sr^{90}+Y^{90}$ in absorbers of different Z and the photons were scattered from magnetized iron. The relative change in counting rate ϵ on reversing the magnetization was measured for different photon energies. The results are represented in Fig. 1. There is no indication of Z dependence

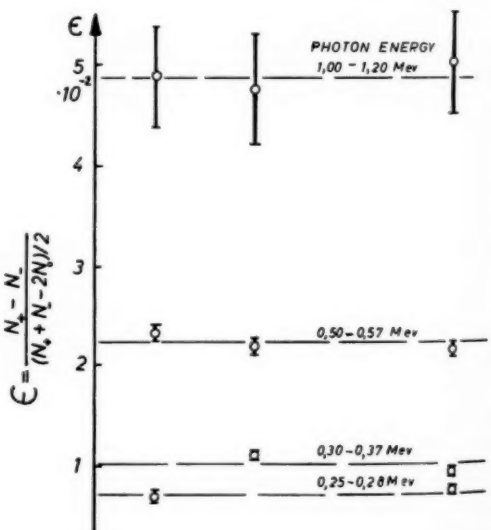

FIG. 1. The relative change in counting rate ϵ on reversal of magnetization as a function of the atomic number Z of absorber. Source: $Sr^{90} + Y^{90}$.

for all photon energies investigated.

This can be explained in the following way: (a) the calculations of Fronsdal and Überall[4] (including screening corrections) show that the fundamental process is Z-independent; (b) the beta particles are very little depolarized by scattering and diffusion in thick absorbers as was shown by Heintze[6]; (c) apparently the polarization of bremsstrahlung is not much affected by electron scattering and diffusion. This seems to be plausible. It is well known that relativistic electrons emit bremsstrahlung preferentially in the forward direction. As a consequence if an electron is deflected by a large angle its bremsstrahlung will not be observed. Furthermore the polarization does not depend markedly on the angle between electron and photon direction.[4] Thus if an electron is deflected by a small angle ($\lesssim 30°$) and emits bremsstrahlung in its former direction of flight the polarization will be practically the same as if the bremsstrahlung was emitted in the forward direction before the deflection took place. From these considerations it may be concluded that the processes happening in the absorber do not change the polarization appreciably. Therefore the experimental results may be compared with calculations for thin absorbers.

However, the electron energy distribution must be taken into account. If one sets the pulse-height discriminator to select photon energies between k and $k+\Delta k$, electrons with kinetic energies between k and E_0 (end-point energy of beta spectrum) will contirbute bremsstrahlung to this energy band and one observes an average polarization $\bar{P}(k)$. This polarization was calculated taking the probability for the emission of a photon with energy k as a function of electron energy and the probability for the emission of a beta particle with energy E as weights. The result for Y^{90} is the full curve in Fig. 2. The experimental points[7] taken from Fig. 1 and previous measurements[2] are in very good agreement with the theoretical curve.

The curve calculated by McVoy[2] for monoenergetic electrons with a kinetic energy of 2 Mev is shown for comparison (dotted curve). It can be seen that this curve is not representative for a beta source. This is of some practical interest as some authors used the external bremsstrahlung for the calibration of polarization analyzers.[8] The broken curve in Fig. 2 indicates the maximum polarization which is produced by electrons with longitudinal polarization v/c in the case of maximum photon energy ($k=E_0$). It was calculated by formula (8) of McVoy's paper[3] multiplying it by v/c. All dotted curves for electron energies different from that shown in Fig. 2 would start on the broken curve and go to the origin. Two measurements for a Tm^{170} source are also shown in Fig. 2. As one expects they lie between the full and the broken curve.

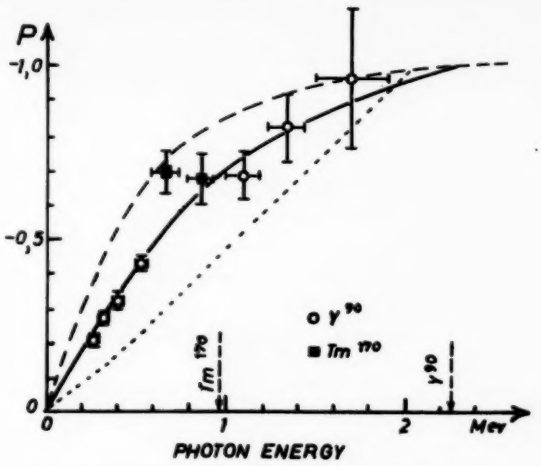

FIG. 2. Circular polarization of external bremsstrahlung as function of photon energy. Full curve: theoretical polarization for Y^{90} taking into account electron energy distribution; dotted curve: theoretical polarization for 2-Mev electrons; broken curve: polarization for the case of photon energy equal to electron energy (electron polarization = v/c). The arrows indicate the end-point energies of the beta spectra.

This investigation shows that the production of circular polarization of bremsstrahlung produced by beta rays in thick absorbers can be understood quantitatively. More information could be obtained by selecting monoenergetic electrons. Such an experiment is under way.

We are indebted to the Bundesministerium für Atomkernenergie und Wasserwirtschaft for supporting this work.

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CIRCULAR POLARIZATION OF EXTERNAL BREMSSTRAHLUNG

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(Received October 10, 1958)

One of the consequences of the nonconservation of parity in β decay is the longitudinal polarization of emitted electrons.[1] It turns out that this polarization gives rise to a circular polarization of the external bremsstrahlung (EB) whose analysis is based on the existence of a spin-dependent part of the Compton cross section.[2] The polarization of EB quanta was observed by a few workers[3-8]; their results indicate that, at least at the high-energy end of the spectrum, the polarization is almost complete.

We have investigated the sense and the degree of the polarization of EB quanta (max energy =2.26 Mev) as a function of their energy and compared the results with a rough theoretical estimation.

The source of β radiation was a silver disk 0.05 mm thick containing uniformly dispersed 40 mC Sr^{90}-carbonate, backed by 0.8 mm of silver and covered by a 0.1-mm layer of silver and by a layer of lead (~1 mm) which absorbed all β rays. Sr^{90} (max energy=0.545 Mev) is in equilibrium with its daughter Y^{90} (max energy =2.26 Mev). We employed a γ polarimeter of the type used by Schopper[7] and by Boehm and Wapstra.[5] The γ rays were scattered off the inner surface of an Armco iron cylinder magnetized to saturation; thus the polarization of γ rays is detected through Compton scattering by polarized electrons. The scattered quanta were detected by means of a 1 1/2-in. × 1-in. NaI (Tl) crystal coupled through a 10-cm Lucite light pipe to an *EMI* 9514 photomultiplier. After amplification, the pulses were fed into a ten-channel pulse-height analyzer. Three iron magnetic shields and a compensating coil were used coaxially with the photomultiplier.

When several corrections are applied, the percentage differences of counting rates for opposite directions of magnetization represent the degree of circular polarization. With this in view we made investigations on the following: (a) energy resolution of the scintillation spectrometer, (b) peak to total ratio for our crystal as a function of γ-ray energy, (c) effective scattering angle (θ=45°), (d) energy spread of the scattered quanta arising from the finite size of the scatterer, (e) contribution of magnetizing coils to scattering, (f) production of low-energy quanta by double scattering in the scatterer, (g) number of polarized electrons/atom ($\nu/Z=7.9\times10^{-2}$). Moreover the efficiency of the polarimeter was evaluated by using the Compton cross sections for polarized electrons[2] and including a correcting factor arising from the finite size of the scatterer.

Figure 1 summarizes the results of our measurements with their statistical error corrected for the effects quoted in (a), (b), (d). The correction ranges from -8% at 0.50 Mev to + 27% at 2.0 Mev; at low energy it is mainly due to the Compton pulse distribution from the crystal, at high energy mainly to the energy spread of the polarimeter. The effect (e) was found negligible. The effect (f), although present in the scattered quanta spectrum, does not distort significantly the measured asymmetries; it was minimized

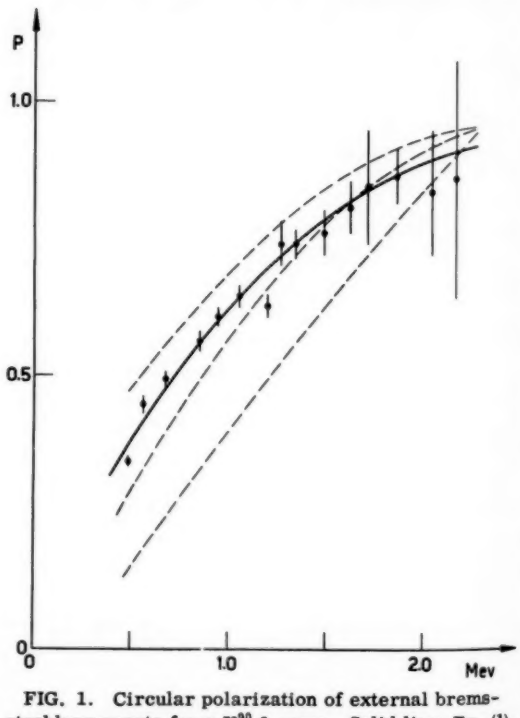

FIG. 1. Circular polarization of external bremsstrahlung quanta from Y^{90} β rays. Solid line: Eq. (1). Dotted lines from top to bottom: ideal undepolarizing matter, silver, and lead.

by a lead conical shield. The measured polarization which was found to be right, fits the equation

$$P=1.06\{1-\exp(-0.90E)\};\ 0.5<E<2.26 \text{ Mev.} \quad (1)$$

Eq. (1) gives for the upper limit of the EB spectrum, $P=0.92$. The estimated error is probably lower than 10%. The limit value, together with the calculation of McVoy,[8] indicates a longitudinal polarization of β rays of $-0.96v/c$ at $v/c = 0.98$.

As regards the polarization of EB quanta at lower energies, we made a comparison with approximate theoretical previsions by assuming (a) that the energy loss of the electrons in the absorber results only from inelastic collisions[9]; (b) that their depolarization results only from multiple electron scattering[10]; (c) that the polarization of EB is proportional to the photon energy irrespective of the energy value of the radiating electron (according to McVoy,[8] that is true for 2-Mev electrons); (d) that the polarization of electrons is equal to $-v/c$ at any energy. Moreover an approximate expression for the EB production cross section, intermediate between that given by Bethe and Heitler[11] and by Elwert,[12] was used and an integration over the β spectrum was performed. The results of these calculations are included in Fig. 1.

A complete account of the present experiment will be published when the investigation in progress on the polarization of EB quanta produced in various absorbers is concluded.

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OBSERVATIONS ON THE PLASMA PRODUCED IN A MAGNETIC MIRROR GEOMETRY

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A xenon plasma has been observed in a device employing the magnetic mirror geometry.[1,2] The apparatus consists of two single-turn magnetic field coils connected through a spark gap switch to a low inductance capacitor bank (25 kv, 14 μf, 0.010 μh). The total inductance of the assembly, including field coils, is 0.04 microhenries with an inherent circuit resistance of 0.006 ohm. The maximum peak current to date has been limited to 225 000 amperes which gives a magnetic field calculated to be approximately 50 000 gauss at the center of each coil. The time for the field to rise to this maximum value is 1.2 microseconds. The separation between the field coils was adjusted to give a magnetic mirror ratio equal to two. No pre-ionization of the gas was employed in making the observations reported here.

The field produced in this geometry has only axial and radial components. When the field is caused to change with time, the former will induce a current into the plasma such that a compression towards the axis will occur. The latter component will cause an expulsion of the gas in the longitudinal direction away from the coils and the region of high magnetic field. Thus if a plasma is accelerated by a rapidly changing field with the above characteristics the result will be two plasma fronts approaching each other from opposite directions while simultaneously collapsing towards the axis.

Such effects have been observed, and typical rotating-drum camera pictures are given in Fig. 1. Figure 1 (a) is the recording of the light output from a slit placed normal to the axis of the discharge tube and 1/8 in. from one field coil. The bright vertical striations are separated in time by one-half the natural period of the condenser bank and are taken to be the passage of radially accelerated plasma fronts through the xenon which becomes excited and ionized. The magnetic field acts upon the ionized gas behind the front causing it to pull away from the walls of the discharge tube as can be seen in the region immediately following the first light intensity maximum. Within this

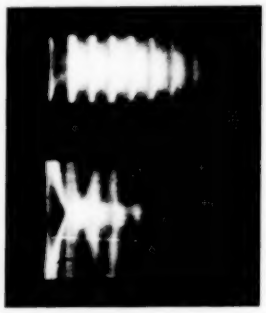

FIG. 1. Streak camera photographs of a xenon plasma produced in the magnetic mirror geometry. The defining slit is oriented perpendicular to the discharge tube axis (a), 1/8 in. from the edge of one field coil, and parallel to the axis (b).

region the plasma is compressed uniformly and shows no evidence of instabilities. Resolution of the succeeding oscillations is impaired due to halation of the film. It is to be noted that the striations become bowed as time increases. This bowing is interpreted to be the resolution in time by the camera of the plasma front as it collapses towards the axis. At earlier times in the discharge the camera is not able to detect the motion since the time taken to reach the axis is less than its resolving time. The bowing of successive striations has been measured for a series of ten exposures as a function of time during the discharge. The normalized velocities derived from these measurements are shown in Fig. 2, the errors being the average deviation from the mean values. Plotted on the same scale is the envelope of the peak voltage amplitudes taken from oscilloscopic data. It is seen that these two curves are coincident within the experimental error. Normalization was required since the exposures were taken over a range of ambient pressures which resulted in different velocities. Depending on the pressure and depending on the intensity maximum being measured, image curvatures have been observed which correspond to velocities ranging from 2.1×10^6 cm/sec to 1.1×10^7 cm/sec. The striations might also be due to the passage of a concave plasma front moving in the longitudinal direction past the transverse slit. This interpretation is believed not to be the case since radial acceleration of the plasma is known to occur.[3] The effect would be particularly noticeable at this position since the radial forces here that are acting on the ionized gas are greater

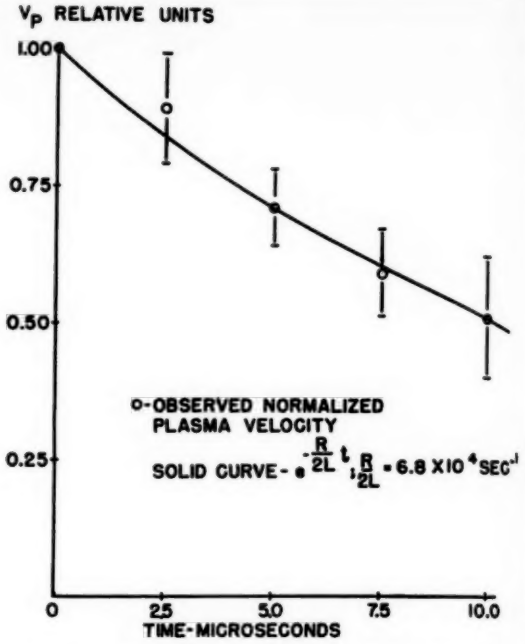

FIG. 2. Normalized plasma velocity vs time.

than the longitudinal forces.

Horizontal propagation of the plasma is shown in Fig. 1 (b) which was chosen to illustrate the collision of the accelerated plasma. The average velocity for the first set of colliding fronts is 1.4×10^6 cm/sec. At lower pressures horizontal velocities up to 7.0×10^6 cm/sec have been recorded. These are average velocities, the front velocity being higher near the coils. The straight vertical lines in this figure are due to reflections from the slit. It is seen that the first collision occurs near the time of zero magnetic field and that reflection is inhibited until after the second field maximum when expansion occurs. This is followed again by successive compressions until the discharge is no longer visible.

The assistance of Mr. J. Saia was greatly appreciated during the construction of apparatus and in the recording of data. We are indebted to Dr. Gordon Milne of the University of Rochester, Rochester, New York, for the loan of the rotating-drum camera. Many helpful discussions were held with Dr. C. K. McLane, Dr. H. N. Olsen, Mr. Saia, and Dr. P. C. Waterman throughout the course of this work.

[1]R. F. Post, Bull. Am. Phys. Soc. Ser. II, 3, 196

(1958).
[2]L. F. Spitzer, Physics of Fully Ionized Gases (Interscience Publishers, New York, 1956), p. 11.
[3]G. S. Janes, Tenth Annual Gaseous Electronics Conference, October, 1957 (unpublished).

IONIZING RADIATION ASSOCIATED WITH SOLAR RADIO NOISE STORM*

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Beginning at 1525 U.T. on 22 August, 1958, ionizing radiation appeared at an atmospheric depth of 10 g/cm^2 in time association with a great solar radio noise storm.[1] The response of three balloon-borne detectors is shown in Fig. 1. The rates of the ion chamber and counter telescope are seen to rise by as much as a factor of ten above the cosmic-ray level while the single counter increases by a much smaller factor. In Table I the relative responses of the three detectors are given. These ratios cannot be obtained with x-rays, high-energy γ rays, or electrons. These ratios are wholly consistent with protons having an energy loss no greater than 4.5 times minimum ionization as was determined by comparing their ion chamber to single counter ratio with that for cosmic rays and using Neher's[2] data on average specific ionization. The above energy loss corresponds to a proton kinetic energy of 170 Mev. With protons of this energy it is readily seen how to account for the ratios of Table I in a qualitative way:

(1) The counter telescope to single counter

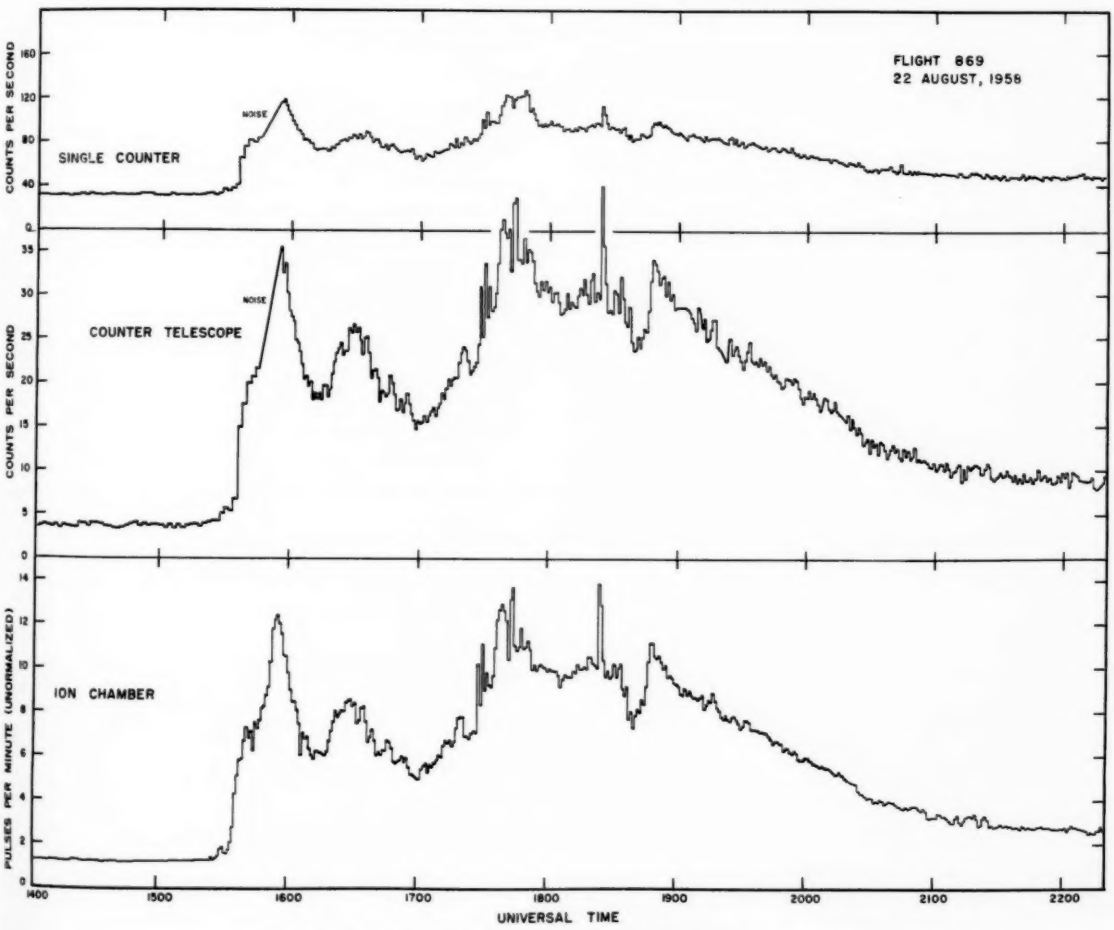

FIG. 1. Counting rates of the three detectors.

ratio should be higher for this energy proton than for cosmic rays due to the strong collimating effect of the atmosphere. Thus the telescope should see a much higher relative increase than the single counter which obtains most of its counts from low zenith angles.

(2) The ion chamber to counter telescope ratio is seen to be about the same as for cosmic rays. It is not higher because the ion chamber is omnidirectional and few protons appear at the lower zenith angles. Although it is seeing relatively fewer particles than the telescope, this is offset by the high specific ionization that protons of this energy possess.

(3) The ion chamber and single counter receive the same relative particle flux increase but since the protons ionize heavily this ratio is considerably larger than for the ordinary cosmic-ray beam.

The extra radiation cannot consist predominantly of alpha particles or other heavy nuclei because the observed energy loss of four times minimum would not be consistent with the strong collimation feature of the radiation. Electrons are ruled out because of the observed very high ion chamber to single counter ratio. For fast electrons this quantity is no higher than that for cosmic rays.

Further information ruling out x-rays was obtained later in the event on a record balloon flight from a sodium iodide scintillation crystal and four-channel kicksorter covering energy losses from 60 to 450 kev. It was found that the total rate of the crystal increased by about a factor of 1.5 while if the extra radiation had been x-rays of the intensity indicated by the ion chamber an increase of several hundred would have been observed.

From one previous balloon flight made at Churchill in 1957, there was strong evidence for the appearance of protons[3] in the 100-Mev energy region during a period of high solar activity but no further confirmation was obtained until the occurence of the present event.

It should be pointed out that the energy determination on the protons made from ion chamber and single counter data and verified by the extent of collimation actually tells little of significance about the proton energy spectrum incident on the atmosphere. This is because the atmosphere above the balloon cuts off the protons completely below about 125 Mev so that if the spectrum is fairly steep the effect produced will correspond to protons of roughly the observed energy.

An interesting feature of the event is the low abundance of electrons and photons. This implies a direct solar origin although the rapid time structure from 1740 to 1900 U.T. seems difficult to account for on this basis alone. These sharp fluctuations might be due to a local modulation effect. The time scale and certain of the details of this event bear a strong resemblance to the x-ray[4] shower observed on August 29-30, 1957, as can be seen from Fig. 2. This suggests that the protons in the present event and the electrons, presumed to give rise to the x-rays in the earlier event, are affected by a common mechanism.

It was definitely established that the radiation present during the August 22, 1958, event was not associated with visible aurorae as sky conditions were excellent for observations. There

Table 1. Relative response of the detectors to solar protons and to cosmic rays.

	Ion chamber to counter telescope ratio	Ion chamber to single counter ratio	Counter telescope to single counter ratio
Cosmic rays prior to 1520 U. T.	5.6×10^{-3}	0.67×10^{-3}	0.12
Extra radiation 1600-1700 U. T.	5.5×10^{-3}	2.0×10^{-3}	0.37
Extra radiation 2000-2100	5.1×10^{-3}	1.9×10^{-3}	0.38

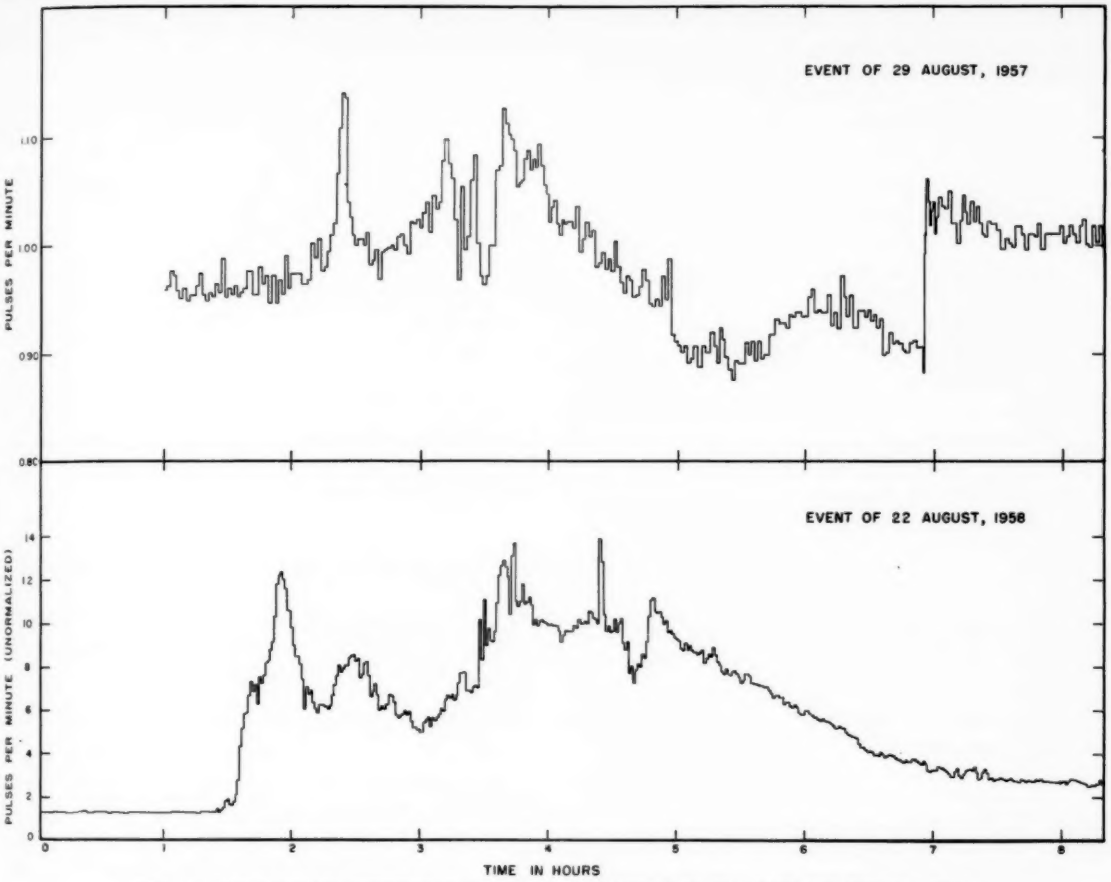

FIG. 2. Ion chamber response to x-ray event of August 29-30, 1957 (top figure), and proton event of August 22-23, 1958 (bottom figure).

was very little if any magnetic disturbance present during this event.

*This work supported by the U. S. National Committee for the International Geophysical Year and the Office of Naval Research.

[1]The author is grateful to Dorothy Trotter of the High Altitude Observatory and to the Canadian Defense Research and Telecommunications Establishment for information on this event.

[2]H. V. Neher, Progress in Cosmic-Ray Physics (North-Holland Publishing Company, Amsterdam, 1952), Vol. 1, p. 261.

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DETERMINATION OF THE π-π INTERACTION STRENGTH FROM π-N SCATTERING*

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Many people have considered the role of the π-π interaction in π-N scattering, particularly in the pion production process.[1] Among the effects which can be qualitatively argued to be a result of the π-π interaction are the following:

(a) The nonresonant behavior of the $i=\frac{1}{2}$ S-wave phase shift at low energies, which indicates a fairly long range for the S-wave interaction.[2]

(b) The positive value of δ_{13} above ~200 Mev,[3] and the positive rise of the $i=\frac{1}{2}$ scattering amplitude in the region 200 to 500 Mev.[4] Taking into account interactions of only the Born approxi-

mation (Tamm-Dancoff), one finds that the P-wave amplitudes are predicted to be negative, and the positive D-wave amplitude is very small up to at least a Bev.[5]

(b′) The large value of the meson production cross section near threshold in the $i=\frac{1}{2}$ state,[6] but not in the $i=\frac{3}{2}$ state. In the latter case the observed production cross section is roughly in agreement with static model calculations, but these are a factor of about 5 too small in the $i=\frac{1}{2}$ state.[7]

(c) The maximum of the total cross section in the $i=\frac{1}{2}$ state at ~0.9 Bev (the "second maximum").[8] It was suggested that this reflected a resonance in the π-π scattering,[9] but it was subsequently pointed out that the spread of momentum of the self-field pions would make the resulting maximum in the π-N cross section very broad.[8] Further, there is no evidence that the final pair of pions tend to have the presumed resonant relative energy. But at this energy the final state resonant π-N interaction is probably important, as well as perhaps "multiple scattering" in the self field, so that the situation is too involved to allow ruling out of this mechanism. For instance, if the π-N resonant final state is indeed important, this may tend to select the momentum of the virtual pion, so that the effective momentum spread is really less.

(d) The large size of the high-energy total cross section, and the consequent backward peaking of the nucleon's angular distribution, in both elastic and inelastic events of low multiplicity.[1] This clearly indicates that the incident π interacts with the virtual pion cloud of the nucleon, rather than with a core.

A way of getting a quantitative estimate of the π-π interaction strength is by isolating the contribution of the one-pion exchange diagram (Fig. 1) from all others. One can attempt to do this by using the fact that this matrix element becomes infinite when the exchanged pion is real, i.e., for a momentum transfer Δ to the nucleon of about $i\mu$. One cannot of course reach this physically, but one can try to extrapolate the cross section as a function of Δ^2 to $\Delta^2 \simeq -\mu^2$. This is just the principle of the threshold theorem, used for instance in the determination of the π-N coupling constant from π-N scattering. Chew[10] has recently suggested the application of the extrapolation procedure to a determination of the π-N coupling constant from N-N scattering. The present case is similar, but in order to deduce the dependence of the matrix element on momentum transfer, one must extract from the cross

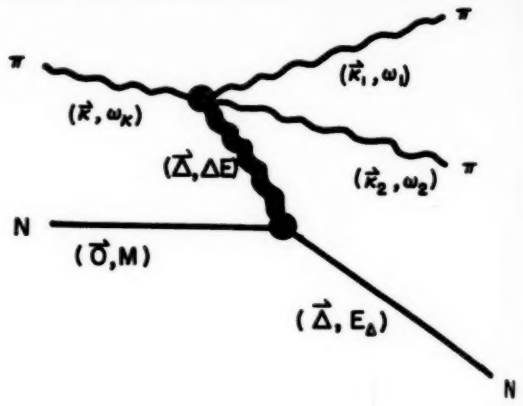

FIG. 1. Feynman diagram of the pole-containing matrix element. Beneath each line is written its momentum and energy.

section the phase space of the three-body final state.[11]

It is convenient to work in the laboratory system. We have, for the differential cross section for single pion production,

$$d\sigma=(2\pi)^4|\mathfrak{M}|^2 d^3k_1 d^3k_2 \delta(\omega_1+\omega_2+E_\Delta-W), \quad (1)$$

where $W=\omega_k+M$ the total energy, and we have put the velocity of the incident pion equal to unity. The pole of $\mathfrak{M}$ occurs at $T_\Delta=-\mu^2/2M$ where $T_\Delta=E_\Delta-M$, the kinetic energy of the final nucleon. It is convenient to describe the final state by the relative momentum of the final mesons (in their center of mass) $\vec{\xi}$ and the total momentum of the mesons $\vec{P}$ or, equivalently, the momentum $\vec{\Delta}$ of the nucleon. The pertinent Jacobian is

$$d^3k_1 d^3k_2 = d^3\xi d^3P \frac{1}{\omega_\xi \Omega_\xi}\left[\Omega_\xi^2 - \frac{(\vec{\xi}\cdot\vec{P})^2}{4\omega_\xi^2}\right],$$

where

$$\omega_\xi^2=\xi^2+\mu^2, \Omega_\xi^2=\omega_\xi^2+P^2/4\,. \quad (2)$$

Integrating over the direction of $\vec{\xi}$ (the length is fixed by energy conservation), we find that Eq. (1) becomes

$$d\sigma=8\pi^5\langle|\mathfrak{M}|^2\rangle v_\xi[(W-E_\Delta)^2-v_\xi^2P^2/3]d^3\Delta, \quad (3)$$

where

$$v_\xi=\frac{\xi}{\omega_\xi}=\left(\frac{\vec{k}\cdot\vec{\Delta}-WT_\Delta-\frac{3}{2}\mu^2}{\vec{k}\cdot\vec{\Delta}-WT_\Delta+\frac{1}{2}\mu^2}\right)^{1/2}. \quad (4)$$

The $\langle\ \rangle$ on $|\mathfrak{M}|^2$ indicate an average. Finally,

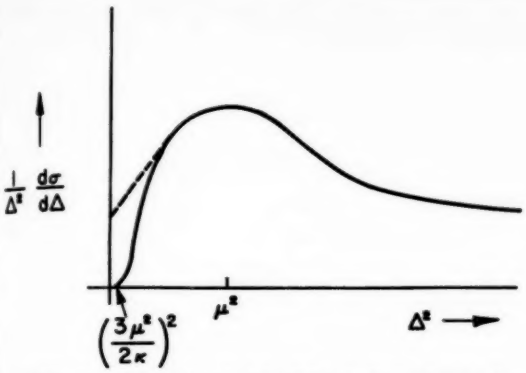

FIG. 2. Schematic sketch of the Δ dependence of the single pion production cross section. The deviation from the dashed line near threshold is the effect of the factor $C(\Delta)$, Eq. (5).

integrating over the direction of $\vec{\Delta}$ we find, for $k \gg \mu$ and $\Delta \ll M$ or k,

$$\frac{d\sigma}{d\Delta} \approx \frac{32}{3}\pi^6 \langle |\mathfrak{M}|^2 \rangle k^2 \Delta^2 C(\Delta) \qquad (5)$$

where

$$C(\Delta) = \frac{(k\Delta - WT_\Delta - \frac{3}{2}\mu^2)^{3/2}}{k\Delta(k\Delta - WT_\Delta + \frac{1}{2}\mu^2)^{1/2}} . \qquad (6)$$

Except very near the threshold, which is at $\Delta \approx \frac{3}{2}\mu^2/k$, the two pions are relativistic in their center-of-mass system, and $C(\Delta) \approx 1$.

The matrix element of the diagram of Fig. 1 (containing the pole) would be of the form

$$|\mathfrak{M}|^2 \sim \Delta^2/(\mu^2+\Delta^2)^2 \qquad (7)$$

for small momenta, whereas other diagrams would contribute matrix elements which are roughly constant; thus the total squared matrix element would look something like Fig. 2. The size of the hump at $\Delta^2 = +\mu^2$ would be a measure of the strength of the π-π interaction.

Of course, the picture is distorted if other contributions to the single-pion production matrix elements interfere strongly with the pole term or have themselves a significant dependence on Δ in the region $\Delta \sim \mu$. The latter possibility is not a priori likely; all indications are that the nucleon is effectively smaller than μ^{-1}. Some evidence on this point is provided by Walker et al.,[12] Fig. 16. This exhibits $d\sigma/d\Delta$, which seems to be proportional to Δ^2 up to $\Delta \approx 500$ Mev/c. They have very few events at small Δ; in fact to investigate our point economically one must ignore events at $\Delta \lesssim 400$ Mev/c, say.

Because of the low energy of the recoil nucleon $\Delta = \mu (\approx 10$ Mev), it seems impractical to do the experiment of measuring $d\sigma/d\Delta$ with counters. The present techniques using diffusion cloud chambers, or perhaps bubble chambers, should be sufficient. One should discriminate against higher multiplicity events, although their inclusion might merely increase the "background" in Fig. 2.

As one goes to higher and higher energies, keeping Δ fixed, one expects that the matrix element of Fig. 1 remains constant (if the π-π cross section does), but all other diagrams become small[13] so that in principle it could be found exactly.

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*This work was supported by the U. S. Atomic Energy Commission.

[1] The most recent experimental papers, which contain references to earlier work, are W. D. Walker, Phys. Rev. 108, 872 (1957); G. Maenchen et al., Phys. Rev. 108, 850 (1957).

[2] See Drell, Friedman, and Zachariasen, Phys. Rev. 104, 236 (1956). With their cutoff $[\omega_{max} = 4.5\mu]$, δ_1 goes through 90° at $k \approx 2.2\mu$. Thus to avoid this one needs to use a "Born approximation" which has a cutoff factor which decreases at energies small compared to M.

[3] See G. Puppi, The Eighth Annual International Conference on High-Energy Nuclear Physics, Geneva (unpublished.

[4] R. Sternheimer, Phys. Rev. 101, 384 (1956). See also reference 7.

[5] Chew, Goldberger, Low, and Nambu, Phys. Rev. 106, 1345 (1957).

[6] W. D. Walker et al., Bull. Am. Phys. Soc. Ser. II, 3, 104 (1958), and private communication from Dr. Walker.

[7] I must thank E. Kazes for drawing this to my attention. His is the latest calculation of pion production in the static model, and contains references to earlier work, Phys. Rev. 107, 1131 (1957).

[8] See, for instance, Cool, Piccioni, and Clark, Phys. Rev. 103, 1082 (1956).

[9] F. J. Dyson, Phys. Rev. 99, 1037 (1955); G. Takeda, Phys. Rev. 100, 440 (1955).

[10] G. F. Chew, University of California Radiation Laboratory Report UCRL-8283 (unpublished).

[11] We average over all final state parameters except Δ in order to "gain statistics" using a given number of events. One would of course learn more details about the π-π interaction if one could get away with more subdivision of the data.

[12] Walker, Hushfar, and Shephard, Phys. Rev. 104, 526 (1956).

[13] That is, except for those corresponding to "diffraction" meson production, in which the diffraction-scattered proton shakes off a meson. E. L. Feinberg and I.Pomeranchuk, Suppl. Nuovo cimento 4, 652 (1956). This cross section is very small.

EXCLUSION PRINCIPLE AND PHENOMENOLOGICAL OPTICAL-MODEL POTENTIALS*

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(Received August 13, 1958)

In recent years the elastic scattering of particles by a nucleus has been successfully analyzed[1] in terms of a simple complex-potential model.[2] In such calculations the exclusion principle has been ignored although for not too energetic bombarding nucleons the effect can be considerable. A modification of the optical model is proposed here which takes into account the effect of the exclusion principle within the framework of a single-particle description of the many-body problem.

Suppose that the nucleus can be described by a single-particle real potential U, which could be momentum as well as coordinate dependent, let the eigenfunctions of $T+U$ be $u_n(r)$, and assume the first N levels are occupied. Suppose further that an incident nucleon in its interaction with the nucleus is to be described by a complex potential V. Note that it is in the nature of the optical model that, although the incident nucleon is identical with those in the nucleus, it is subject to a different complex potential to simulate the inelastic processes. Let the incident nucleon be described by the wave function ψ. We seek a one-body description of the scattering. Consequently, if we ignore the exclusion principle and let T be the kinetic energy, we are led to the Schrödinger equation

$$(T+V)\psi = E\psi. \tag{1}$$

It is clear that the solutions to Eq. (1) are in general not orthogonal to the wave functions $u_n(r)$, for $n=1, 2 \ldots N$, describing the nucleons in the nucleus. Let Λ be a projection operator onto the N occupied states. Then Eq. (1) should be replaced with

$$[T+U+(1-\Lambda)(V-U)]\,\psi = E\psi. \tag{2}$$

The solutions ψ of this equation are orthogonal to the occupied states. Note that the orthogonal complement of the space spanned by the occupied states is not invariant under the operator $T+V$. That is why it would not be correct simply to solve Eq. (1) and then to project out the occupied states. If the occupied states are eigenfunctions of $T+V$, then Eq. (2) reduces to Eq. (1). The effect of the exclusion principle is then taken into account simply by choosing orthogonal solutions to the Schrödinger equation.

Clearly Eq. (2) is a simple extension of the work of Bethe and Goldstone.[3] One of us[4] has been able to show rigorously, starting from the n-body problem, that the elastic scattering of a nucleon by a nucleus can be described by an equation of the form of Eq. (2), with V a nonlocal potential. Furthermore, an explicit expression was given for the potential in terms of the assumed forces between nucleons.

The point of this note is to suggest the use of Eq. (2) with V as a phenomenological potential. Actually, the exclusion principle will also have an explicit effect on V, arising for example from the requirement that in intermediate states the target nucleon as well as the incident nucleon must be in unoccupied states. We do not know the additional restrictions this may impose on V, but nevertheless suggest that Eq. (2) is clearly more nearly correct than Eq. (1).

If V is taken to be a local potential, the solutions to Eq. (2) are not only quantitatively but also qualitatively different from the solutions to Eq. (1). This may be seen by considering the one-dimensional problem in which a particle of mass m and energy E impinges on a potential hole

$$\begin{aligned} V(x) &= 0, \quad x<0, \quad x>t, \\ &= -V_0 - iV_1, \quad 0<x<t, \end{aligned} \tag{3}$$

while a single identical particle is confined in the lowest state of an infinitely deep potential hole of the same extent. For this problem Eq. (2) can be solved exactly, and taking the limit of large t leads to the following simple expressions for the transmitted and reflected amplitudes if the incident wave has unit amplitude:

$$\begin{aligned} T &= -\frac{e^{-ikt}}{(k+\alpha)^2}\left[4k\alpha e^{i\alpha t} + \frac{4\pi^2 ki}{t^3\,\alpha^2}\right], \\ R &= \frac{1}{(k+\alpha)^2}\left[(k^2-\alpha^2) + \frac{4\pi^2 i}{t^3\,\alpha}\right], \end{aligned} \tag{4}$$

where

$$k^2 = 2mE/\hbar^2,$$
$$\alpha^2 = (2m/\hbar^2)[E+V_0+iV_1],$$
$$\mathrm{Im}\ \alpha > 0.$$

The first term in each of these two expressions is the usual result while the second term re-

flects the effect of the exclusion principle. The transmission is thus greatly increased, since the first term in T corresponds to an exponential decay of ψ due to absorption. The relative change in R is much smaller. The t dependence of the exclusion-principle terms originates in part from the fact that the amount of momentum space occupied, and therefore excluded, by the bound nucleon decreases with increasing potential width.

This example indicates that Eqs. (1) and (2), with the same potential V, will in general lead to different angular distributions and total cross sections. We want to emphasize the corollary: the phenomenological potential needed in order to fit given experimental data will depend on whether Eq. (1) or Eq. (2) is used. It might be felt that in a phenomenological calculation this is unimportant since V is adjusted to fit the experiments and is used only to summarize a great many data in a convenient form. We do not feel this is a valid view for two reasons.

First, a great deal of physical significance has always been ascribed to the optical parameters—such as a nuclear radius, and surface thickness. Including the exclusion principle will inevitably lead to different optical parameters. In principle, presumably, there always exists a "potential" which fits any given scattering data by means of Eq. (1); in practice only energy-dependent and spin-orbit, but otherwise local, potentials have been used. The effect of a local potential in Eq. (2) is quite different, since the exclusion principle already acts in a highly nonlocal manner. It is therefore to be expected that the use of a local potential in Eq. (2) has more physical significance than in Eq. (1).

Second, from a theoretical point of view, any fundamental theory attempting to describe the scattering of a nucleon by a nucleus in terms of nucleon-nucleon forces[4] will lead to an equation of the form of Eq. (2). If the analysis of data in terms of optical parameters is to prove useful to the theoretician, i.e., remove the necessity for theoretical calculations of cross sections or phase shifts, then the optical parameters must refer to Eq. (2).

Since Eq. (2) is nonlocal there may be some concern over the requirements of causality. We are unaware of any sufficiently general expression of this requirement to be applicable to this equation.[5] Although Eq. (2) does not lead to a local current continuity equation, it can be easily shown that the wave function normalization will decrease monotonically for a potential V whose imaginary part is everywhere negative.

In conclusion, we observe that the numerical calculations of scattering with Eq. (2) are not essentially more difficult than with the Schrödinger equation. One starts by calculating the first N states of U. Then recalling that

$$\Lambda(\vec{r},\vec{r}') = \sum_{n=1}^{N} u_n(\vec{r})\, u_n^*(\vec{r}'),$$

Eq. (2) may be written in the form[6]

$$(T+V-E)\,\psi(\vec{r}) = \sum_{n=1}^{N} \lambda_n u_n(\vec{r}), \qquad (5)$$

where the λ_n need only be chosen so that ψ is orthogonal to the functions u_n for $n=1,2\ldots N$. The solution of the inhomogeneous equation for ψ is straightforward for arbitrary λ_n, and finally the constants λ_n must be evaluated by the orthogonality requirements. An explicit expression for λ_n in terms of Green's function of the problem without the exclusion principle is easily derived, but does not seem very useful from a practical point of view.

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* Supported in part by the National Science Foundation.

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[5] N. G. van Kampen [Phys. Rev. 91, 1267 (1953)] has considered the requirements imposed on the scattering by causality and found them satisfied by local potentials of finite range. However, even local potentials which decrease at infinity like exponentials may violate his causality criterion. Such potentials, which may lead to poles of the S matrix anywhere in the first quadrant of the complex k plane, can be easily constructed by the Bargmann procedure [V. Bargmann, Revs. Modern Phys. 21, 488 (1949)].

[6] This equation is mathematically equivalent to those used by Sachs [R. G. Sachs, Phys. Rev. 95, 1065 (1954)] for what he called orthogonality scattering. The physics involved, however, is quite different.

NEW EXPERIMENTAL TEST OF SPECIAL RELATIVITY

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(Received September 29, 1958)

The relative frequency stability of two beam-type maser oscillators is used to test the dependence of the velocity of light on velocity of the frame of reference with considerably more precision than has been obtained from experiments of the Michelson-Morley[1] type. Expressed in terms of an ether, the maximum ether drift is shown to be less than 1/1000 of the earth's orbital velocity.

The experiment, which was performed at the Watson Laboratory, involves comparison of the frequencies of two masers[2] having their beams of NH_3 molecules traveling in opposite directions. Møller[3] has analyzed this case and given the change in frequency of a beam-type maser due to ether drift, assuming the molecules in the beam to have a velocity u with respect to the cavity through which they pass, and the cavity to have a velocity v with respect to the ether. The shift may be simply discussed by assuming that, if v is zero, radiation is emitted perpendicularly to the molecular velocity so that there is no Doppler shift. If the cavity and beam are then transported at velocity v through the ether in a direction parallel to u, radiation must be emitted by the molecules slightly forward at an angle $\theta = \pi/2 - v/c$ with respect to u. The fractional change in frequency due to the Doppler effect is then $\epsilon = u/c \cos\theta$ or uv/c^2 due to motion through the ether, assuming that the proper molecular frequencies are unchanged by such motion.

For a thermal molecular velocity of 0.6 km/sec and for the earth's orbital velocity (30 km/sec), $\epsilon = 2 \times 10^{-10}$. The difference in frequency due to the above effect between two masers with oppositely directed beams would be $2\epsilon\nu$, or about 10 cps for ν equal to 23 870 Mc/sec, the NH_3 inversion frequency.

Although uv/c^2 is of second order in the velocities, it is of first order in the velocity of the cavity, or of the laboratory, with respect to the ether. The present experiment measures the entire effect with a rather small fractional error, which affords a particularly small upper limit to v since this quantity enters in first order, rather than in second order as in the Michelson-Morley experiment. A somewhat similar term would occur in the latter experiment if the interferometer used were transported by a plane of speed u, and interference fringes were compared for two opposite directions of flight.

Two maser oscillators with oppositely directed beams were mounted with necessary auxiliary equipment on a rack which could be rotated about a vertical axis. The beat frequency between the two oscillators was adjusted to about 20 cps and recorded continuously. After approximately one minute of recording with the maser axes oriented in an east-west direction, the apparatus was rotated 180° and the beat frequency recorded in the new position.

The change in beat frequency, on the basis of an ether drift, should be $4\epsilon\nu$, or about 20 cps. Sixteen such comparisons were made during a period of about 20 minutes. These were repeated about once per hour during a time somewhat longer than 12 hours, so that the earth's rotation would sweep the east-west direction through a plane.

A relative change in frequency of the two oscillators amounting to about 1 cps was found when they were rotated through 180°. This change is largely due to the earth's magnetic field and other local magnetic fields from which no shielding was attempted. The significant observation is that this change was independent of the time of day (or orientation of the earth), as indicated in Fig. 1.

The first series of measurements was made during a week-day, when local magnetic fields and line voltages were varying. It showed some systematic variations in the effect measured as large as $\pm 1/20$ cps during the day. A second series of measurements, taken on a Saturday when local disturbances were less serious, showed no variation greater than $\pm 1/50$ cps as indicated in Fig. 1, and even these appear random and not simply correlated with time (or the earth's orientation). This precision corresponds to a comparison of frequencies of the two masers to one part in 10^{12}.

The results show that any term of the form uv/c^2 must be smaller by a factor of at least 1000 than what would be predicted by setting v equal to the earth's orbital velocity. That is, velocity with respect to an ether in a plane per-

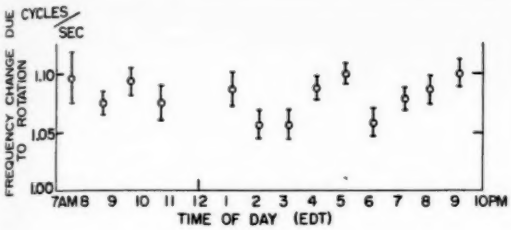

FIG. 1. Diurnal variation of the change in relative frequency due to rotating two ammonia masers through 180°.

Beams of the two masers were oppositely directed and in an east-west direction. The change of about 1.08 cps is primarily due to local magnetic fields. Maximum deviation from this value during the day is 1/50 cps. Lengths of lines indicate probable errors computed from fluctuations of 16 measurements at each point.

pendicular to the earth's axis must be less than 1/30 km/sec. Results from experiments of the Michelson-Morley type vary from an ether drift of about 8 km/sec reported by Miller[4] to an upper limit of 1.5 km/sec given by the experiments of Joos.[5] Of course a major part of the advantage of the present experiment is its first-order rather than second-order dependence on v.

Those who are already completely convinced of the correctness of special relativity, or who do not wish to consider an ether model, should note that postulates of special relativity are not necessarily inconsistent with the existence of a frequency shift in the above experiment or of an anisotropy in space. These can result from the presence of matter external to the earth which is not uniformly distributed, or which is not moving with the earth's velocity.

The preliminary results quoted here apply to September 20, 1958. It is expected that the experiment will be refined further and that additional measurements will be made at other times during the year.

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ANISOTROPY OF THE C^{13} CHEMICAL SHIFT IN CALCITE

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(Received October 10, 1958)

It has been shown by Ramsey[1] that the chemical shift of a nuclear magnetic resonance can be anisotropic. Since the shift tensor is a dyadic, the shifts will be a function of the orientation of the molecule in a magnetic field for nuclei in environments of lower than tetrahedral symmetry. An indication of the presence of such an anisotropy was first found by Bloembergen and Rowland,[2] who attributed the asymmetry of the Tl^{205} resonance in powdered Tl_2O_3 to this effect. Shift anisotropies have also been invoked by Gutowsky and Woessner[3] to explain the T_1 difference between H^1 and F^{19} in 1, 3, 5-trifluorobenzene, and by McConnell and Holm[4] to account for the relatively short T_1 of C^{13} in CS_2.

A direct measurement of the anisotropy of the C^{13} shift in a single crystal of calcite ($CaCO_3$) has now been made. This substance is ideal for such an experiment because all magnetic nuclei are present in such low abundance that dipolar broadening is negligible and a sharp strong line is observed. The apparatus used was a Varian 4300B High Resolution NMR spectrometer operating at 8.5 Mc/sec, and a Varian 12-inch electromagnet. The dispersion mode was used and the lines were measured under rapid passage conditions. The observed line widths, about 20 milligauss, were the result of H_1 broadening. The magnetic field inhomogeneity over the sample volume was about 10 milligauss, and approximate calculations by the method of Kittel and Abrahams[5] indicated a dipolar broadening of the order of 5 milligauss. T_1 was found to be about 40 minutes in the sample used, which was a cleavage rhombohedron of clear colorless "Iceland spar" about 0.3 cm^3 in volume. The crystal was aligned visually with its trigonal axis in the plane of a graduated glass ring which was set vertically in a tube filled with acetone, whose C^{13} resonances were used as secondary standards. The whole assembly was turned by a goniometer head attached to the probe. Final alignment was made by observation of the angular variation of the shift and was accurate to within about one degree.

The carbonate ion (CO_3^{--}) has a three-fold axis. The operation of the shift dyadic will, therefore, give a shift ellipsoid of revolution which can be

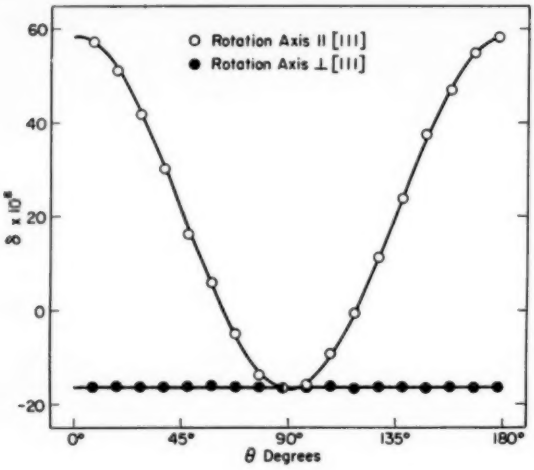

FIG. 1. The angular variation of the C^{13} chemical shift in calcite. The closed circles are for rotation about the trigonal axis, and the open circles are for rotation about an axis perpendicular to the trigonal axis.

completely specified[6] by

$$\delta = \delta_1 \cos^2\theta + \delta_2 \sin^2\theta, \qquad (1)$$

where θ is the angle between the three-fold axis and the magnetic field direction. The experimental results are shown in Fig. 1, along with a plot of Eq. (1) with the parameters $\delta_1 = 58.4 \times 10^{-6}$ and $\delta_2 = -16.6 \times 10^{-6}$. The zero of reference is the shift of the carboxyl carbon in acetic acid and the shift is defined by

$$\delta = \frac{H_{\text{sample}} - H_{\text{reference}}}{H_{\text{reference}}}. \qquad (2)$$

The average shift, $\delta_{Av} = 1/3(\delta_1 + 2\delta_2)$, is 8×10^{-6}, about the same as the CO_3^{--} shift in an aqueous solution of K_2CO_3 (7×10^{-6}).[7] The total anisotropy, 75.0×10^{-6}, is about one-fourth the total range of shifts observed in carbon compounds.[7,8]

Anisotropy measurements such as those described here obviously require single crystals dilute in both the observed nuclear species and other magnetic nuclei. Carbon compounds satisfy the first criterion almost ideally, but the second is much more restrictive. The new technique of line narrowing by double resonance[9,10] would seem to offer the possibility of extending the study of anisotropies to many other substances. The only other crystal in which we have observed an anisotropic shift to date is aragonite, the orthorhombic form of calcium carbonate.

*Multiple Fellowship on Silicones sustained by the Corning Glass Works and Dow Corning Corporation.

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ABSTRACTS

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in THE PHYSICAL REVIEW. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

MICROWAVE PROPAGATION IN HOT MAGNETO-PLASMAS. J. E. Drummond, Poulter Laboratories, Stanford Research Institute, Menlo Park, California (Received April 9, 1958).

The indices of refraction of circularly polarized microwave signals propagating along the magnetic field direction within a high-temperature ionized gas are computed by use of a new conductivity tensor. It is found that phase changes and Landau diffusion damping resulting from electron drift become important for certain ranges of the parameters. These effects are enhanced by the resonance that occurs near the electron cyclotron frequency. The refractive indices depend sensitively upon electron density and temperature.

HYDROMAGNETIC EQUATIONS FOR TWO ISOTOPES IN A COMPLETELY IONIZED GAS. Joseph Slepian, 1115 Lancaster Street, Pittsburgh, Pennsylvania (Received June 9, 1958).

The hydromagnetic equation for a completely ionized gas of high density with a single isotope is given, and from it the two hydromagnetic equations with two isotopes are derived. The difference between the accelerations of the isotopes at any internal point is then equal to $\delta M/M$ times $(1/\rho)[\vec{j}\times\vec{B}]$, the force on the induced current per unit density due to the magnetic field.

It is then shown that the ionic centrifuge gives almost no enrichment of the isotopes deposited, whereas the magneto-ionic expander gives a very high enrichment.

FUNDAMENTAL ABSORPTION EDGE IN CADMIUM SULFIDE. David Dutton, Institute of Optics, University of Rochester, Rochester, New York (Received June 23, 1958).

The absorption and reflection spectra of CdS have been determined in the temperature range 90°-340°K by photoelectric measurements on single crystals, using polarized light. The temperature and frequency dependence of the absorption coefficient in the edge, over a substantial range of absorption magnitudes, are well described by expressions of the form $\alpha(\nu)=\alpha_0 \times \exp[-\beta(E_{00}-CT-h\nu)/kT]$, where the energy $(E_{00}-CT)$ is closely associated with the position of an absorption peak. For light polarized with E vector parallel to the crystalline c axis, there is one such peak, at $\lambda=4844$A at 90°K; for light polarized with E vector perpendicular to the c axis, the 4844 absorption is again present in about equal strength and a stronger line appears at $\lambda=4874$A, shifting the absorption edge toward longer wavelengths by a corresponding amount. These absorption lines, or sharp bands, are observed photographically by Gross and others. These results are discussed in the light of the present theoretical picture of absorption in insulating crystals, and it is noted that an exponential absorption edge of this kind, characteristic also of other ionic cyrstals, cannot be adequately explained in terms of existing models.

MECHANICAL RESONANCE DISPERSION IN QUARTZ AT AUDIO-FREQUENCIES. Edwin R. Fitzgerald, Department of Physics, The Pennsylvania State University, University Park, Pennsylvania (Received June 27, 1958).

Measurements of the complex shear compliance ($J^*=J'-iJ''$) of single crystals of quartz and fused quartz at frequencies from 100 to 5000 cps have resulted in the discovery of sharp resonances in the compliance similar to those recently found in polycrystalline metals and crystalline polymers. The number, locations, and magnitudes of the resonances depend on crystal orientation with respect to the applied dynamic stress and vary with temperature, external static stress, and, in some cases, with time. The presence of numerous resonances in fused quartz may result from the existence of regions of long-range order (100 to 200 A) in this material. Analysis of the data on the basis of a generalized stress-strain relation involving a linear combination of strain and its first and second derivatives gives a close fit to the experimental curves. An explanation of the resonances is suggested by calculations of Fermi, Pasta, and Ulam for nonlinear systems in which no tendency toward equipartition of energy among modes was found. Accordingly it is proposed that: (1) crystalline solids with nonlinear forces between atoms do

not share their vibrational energy among all of the available modes, but pass energy back and forth among relatively few modes; (2) the frequency of the energy exchange among modes may be low, and, in particular, much lower than the frequencies of the lattice vibrations; and (3) the resonance dispersions observed in the dynamic mechanical compliance of quartz and other materials occur at these various acoustic exchange frequencies. This lack of energy equipartition will not necessarily be noticed in specific heat measurements on solids, but can be expected to have some consequences in other areas.

MAGNETIC STRUCTURE OF Fe_4N. B. C. Frazer,* Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received July 15, 1958).

The magnetic structure of Fe_4N has been examined in a neutron diffraction study. The results are in agreement with a model proposed by Wiener and Berger on the basis of magnetic measurements on a series of related compounds. Ferromagnetically aligned moments of $3\mu_B$ and $2\mu_B$ are found for the corner and face-center Fe atoms (respectively) in the cubic unit cell. The difference in moments is apparently due to bonding interaction between nitrogen, at the body-center position, and the face-center Fe's.

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THEORY OF THE HALL EFFECT IN FERROMAGNETICS. J. M. Luttinger, Ecole Normale Supérieure, Laboratoire de Physique, Paris, France (Received July 2, 1958).

The Hall Effect in ferromagnetic substances is computed on the basis of a simple model, making use of the transport theory of Kohn and Luttinger. The calculation is rigorous, but assumes a slowly varying scattering potential, a simple band, and very few conduction electrons. None of these assumptions are very realistic for a true ferromagnetic, but we are interested here only in giving a discussion of the types of contributions which can occur. Terms related to those previously found by Smit, by Karplus and Luttinger, and some new ones, are found. In addition, some comments of Smit on the general problem of the ferromagnetic Hall Effect are discussed.

LOW-TEMPERATURE HEAT CAPACITY OF PURE AND REDUCED RUTILE. P. H. Keesom and N. Pearlman, Department of Physics, Purdue University, Lafayette, Indiana (Received July 14, 1958).

The heat capacity of pure and reduced rutile has been measured between 1 and 20°K. Below 4°K, C of pure rutile is proportional to T^3 with a Debye characteristic temperature θ equal to 758°K. Upon reduction a large additional contribution appears which is independent of temperature below 13°K. This is probably due to electrons which do not become degenerate above 1°K because of their high effective mass (300 - 500 times the free-electron mass).

HIGH-VACUUM STUDIES OF SURFACE RECOMBINATION VELOCITY FOR GERMANIUM. H. H. Madden and H. E. Farnsworth, Barus Research Laboratory, Brown University, Providence, Rhode Island (Received March 3, 1958).

The surface recombination velocity, s, of minority carriers for (100) faces of germanium crystals has been studied under various vacuum conditions. Values of mean lifetime were obtained by the decay-of-photoconductivity method and the corresponding values of surface recombination velocity were determined from geometrical considerations. Increases in s were observed due to initial evacuation of the experimental tube. Following outgassing of the samples in vacuum at temperatures above 500°C and for a time in excess of 30 hours, s was found to be insensitive to exposure to oxygen at pressures as high as 10^{-4} mm Hg. As a result of cleaning the surfaces of samples by the ion-bombardment technique, s increased to a value of $(5 \text{ to } 7)\times10^3$ cm/sec. Heating the ion-bombarded crystals in vacuum at temperatures above 500°C for prolonged periods of time, followed by slow cooling, resulted in a decrease in s to a value below 500 cm/sec but greater than the value obtained following a *CP*-4 etch and the value obtained after pumping. The recombination velocity of clean and annealed (100) germanium surfaces was found to be insensitive to room-temperature adsorption of oxygen at pressures as high as 5×10^{-5} mm Hg. Exposure of these clean sur-

faces to oxygen at temperatures of about 100°C resulted in small decreases in mean lifetime, corresponding to a possible increase in s, assuming that this was entirely a surface phenomenon. The clean-surface value of s could be restored by heating at 500°C in vacuum for short periods of time. The characteristics of a well outgassed oxide-covered surface and a clean surface are found to be similar. The observed similarities are discussed and the conclusion drawn that the oxide layer, of itself, has little or no effect on the surface recombination process.

ANALYSIS OF GALVANOMAGNETIC DE HAAS-VAN ALPHEN TYPE OSCILLATIONS IN GRAPHITE. D. E. Soule, Research Laboratories, National Carbon Company, Cleveland, Ohio (Received June 30, 1958).

An analysis has been made of oscillations in the Hall effect and magnetoresistance for graphite single crystals at 4.2°K with the field parallel to the hexagonal axis. Two periods of 2.11×10^{-5} gauss^{-1} and 1.58×10^{-5} gauss^{-1} are shown to be due to the majority electrons and holes, respectively. These same two values were found in both galvanomagnetic effects and are in reasonable agreement with those observed in the susceptibility. There is a phase difference of π between the two galvanomagnetic properties. An analysis of the magnetic field dependence of the amplitudes incorporating both effects in a "galvanomagnetic ratio," ρ/R, has been made giving effective mass values of 0.030 m_0 for the electrons and 0.060 m_0 for the holes. These are in substantial agreement with those calculated from cyclotron resonance and from the temperature dependence of the susceptibility de Haas-van Alphen oscillations. Corresponding Fermi energies were found to be 0.018 ev for the electrons and 0.012 ev for the holes giving a very slight band overlap in graphite of 0.030 ev.

MAGNETIC FIELD DEPENDENCE OF THE HALL EFFECT AND MAGNETORESISTANCE IN GRAPHITE SINGLE CRYSTALS. D. E. Soule, Research Laboratories, National Carbon Company, Cleveland, Ohio (Received June 30, 1958).

The Hall coefficient and magnetoresistance in purified (99.995%) natural graphite single crystals have been measured from 25 to 25 000 gauss at 298°, 77°, and 4.2°K with the field oriented parallel to the hexagonal axis. Special care was taken in the micromanipulation and strain-free mounting of these small soft crystals. Fast minority carriers due to Fermi surface warping were discovered by their effect on the low-field Hall coefficient behavior. The compensating effect between the majority electron and hole densities [$(5\text{-}2)\times10^{18}$ cm^{-3} with $n_e/n_h = 1.0 - 1.15$] and mobilities [$(1.5\text{-}130)\times10^4$ cm^2/v sec with $\mu_e/\mu_h = 1.10\text{-}0.79$] makes the Hall coefficient very sensitive to the temperature, impurities, and field where it even changes sign. A quadratic low-field room-temperature magnetoresistance dependence progresses at higher fields to an impurity insensitive $H^{1.78}$ behavior. The large magnetoresistance ratio of graphite (~10^5 at 4.2°K and 23 kilogauss), along with the appearance of de Haas-van Alphen type oscillations in these properties, demonstrates the small effective masses ($0.03m_0$, $0.06m_0$) and long relaxation times (2.5×10^{-11} sec at 4.2°K). The mobility follows a $T^{-1.2}$ law in the lattice scattering region $\gtrsim$ 50°K. Low-temperature results, showing carrier density differences and mobilities to be most sensitive to impurities, substantiate the relatively high purity of these crystals.

ANALYSIS OF MULTICARRIER GALVANOMAGNETIC DATA FOR GRAPHITE. J. W. McClure, Research Laboratories, National Carbon Company, Cleveland, Ohio (Received June 30, 1958).

The magnetic-field-dependent data of Soule for the Hall effect and magnetoresistance in graphite have been analyzed using a multicarrier model. An improved mode of analysis is used, in which the magnetoconductivity tensor elements are computed as functions of magnetic field strength from experimental data, and then fitted to simple formulas. The formulas represent solutions to the Boltzmann equation in the classical (non-oscillatory) range. The effects of electrons and holes are separated by applying a Kramers-Kronig type relation. The results, which agree with band model predictions within 20 to 50%, are that there are 2.9×10^{18} holes and electrons per cm^3 in pure graphite at 4.2°K, and 7.0×10^{18} cm^{-3} each at 300°K. The mobilities range from about 9×10^5 cm^2/volt sec at 4.2°K to 1.0×10^4 cm^2/volt sec at 300°K, with the hole-to-electron mobility ratio being 1.2 and 0.9 at the two temperatures. In addition, at room temperature there are about 6×10^{14} minority holes per cm^3

with a mobility of 15×10^5 cm^2/volt sec and 5×10^{14} minority electrons per cm^3 with a mobility 4×10^5 cm^2/volt sec. The relaxation times for the majority carriers are distributed over a range of a factor of four. The average relaxation times are consistent with those deduced from cyclotron resonance experiments.

VELOCITY OF SIDEWISE 180° DOMAIN WALL MOTION IN $BaTiO_3$ AS A FUNCTION OF THE APPLIED ELECTRIC FIELD. Robert C. Miller and Albert Savage, Bell Telephone Laboratories, Murray Hill, New Jersey (Received July 3, 1958).

New techniques are described which permit one to reverse the direction of the spontaneous polarization in liquid electroded single crystals of $BaTiO_3$ with one growing domain which nucleates at a predetermined location on the crystal and then expands through sidewise 180° domain wall motion until the spontaneous polarization in the entire electroded area is reversed. Measurements of the rate of growth of the single domain for a range of applied electric fields give rise to the first quantitative data on the field dependence of the sidewise 180° domain wall velocity in $BaTiO_3$. It is found that this wall velocity is given by $v=v_\infty e^{-\delta/E}$ where δ is several thousands of volts per centimeter and v_∞ is in general of the order of 10 cm sec^{-1}. The fields employed in these measurements were in the neighborhood of several hundreds of volts per centimeter and the wall velocities varied from about 10^{-6} cm sec^{-1} to 10^{-2} cm sec^{-1}. The measured field dependence of the wall velocity is indicative of a nucleation mechanism for the wall motion, and it is shown that the equation of motion usually employed for domain boundaries does not describe the sidewise 180° wall motion in $BaTiO_3$. In addition, the well known $e^{-\alpha/E}$ polarization reversal characteristics of metal electroded cyrstals are shown to be consistent with wall motion of the type found in this research.

INVERSION SYMMETRY OF *M* AND *R* CENTERS. A. W. Overhauser* and H. Rüchardt†, Department of Physics, Cornell University, Ithaca, New York (Received July 7, 1958).

The energy states of color centers lacking inversion symmetry will generally possess permanent dipole moments. Consequently the optical absorption bands of such centers will exhibit linear Stark effects and should become dichroic in large electric fields. No dichroism for the *M* or *R* bands in NaCl or KCl was observed at 78°K as a result of applied fields of 3×10^5v/cm. Since the sensitivity of the measurement was several hundred times greater than that required to detect the field-induced dichroism calculated for a model lacking inversion symmetry, it is tentatively concluded that *M* and *R* centers possess inversion symmetry.

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DYNAMICAL THEORY OF DIFFUSION IN CRYSTALS. Stuart A. Rice, Department of Chemistry and Institute for the Study of Metals, University of Chicago, Chicago, Illinois (Received December 19, 1957; revised manuscript received May 12, 1958).

The theory of self-diffusion in a crystal via the vacancy mechanism is investigated from a detailed dynamical model. It is shown that the parameters which determine the diffusion coefficient can be defined in terms of the normal coordinates of the crystal. The effects of lattice imperfections are considered explicitly in the formulation in the normal mode analysis, but no detailed analysis is given. The effects of the correlation in motion of atoms is considered and some comments on the physical nature of the contributions to the enthalpy and entropy of activation are presented.

MAGNETORESISTANCE IN PbS, PbSe, AND PbTe AT 295°, 77.4°, and 4.2°K. Robert S. Allgaier, U. S. Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland, and University of Maryland, College Park, Maryland (Received June 4, 1958).

Magnetoresistance measurements made on 14 single crystals of PbS, PbSe, and PbTe at 77.4°K conformed to the general phenomenological small-field theory; i.e., the magnetoresistance was proportional to H^2, and varied sinusoidally with the angle between the sample current and magnetic field. Because of the extremely high mobilities in the samples at 4.2°K, small-field theory was generally not applicable, and saturation effects and striking deviations from the sinusoidal behavior were observed at this tem-

perature. The longitudinal magnetoresistance was generally larger (in some cases, 4 or 5 times larger) than the transverse at both 77.4° and 4.2°K. The results did not appear to be significantly different for n- and p-type material. The data generally favor a model with considerable mass anisotropy; in particular, the many-valley model having ellipsoids of revolution along the ⟨111⟩ directions, with a ratio of longitudinal to transverse effective mass of between 4 and 6, seems most appropriate of PbTe.

SPECIFIC HEAT OF A FACE-CENTERED CUBIC Mn-Fe ALLOY AT LIQUID HELIUM TEMPERATURES. Chuan-Tseng Wei, Chin-Huan Cheng and Paul A. Beck, University of Illinois, Urbana, Illinois (Received July 11, 1958).

An Fe+45 atomic percent Mn alloy, vacuum melted and homogenized at 1050°C for 48 hours, was found to consist at room temperature of a single face-centered cubic phase, even after an excursion to liquid helium temperatures. Low-temperature specific heats were determined in the range from about 1.7 to 4.8°K. The electronic specific heat coefficient was found to be 14.6×10^{-4} cal mole $^{-1}$deg^{-2}. The Debye temperature was θ=482°K.

ALPHA-PARTICLE IRRADIATION OF Ge AT 4.2°K. G. W. Gobeli, Department of Physics, Purdue University, Lafayette, Indiana (Received July 14, 1958).

Degenerate n- and p-type germanium samples were irradiated at 4.2°K with polonium alpha particles. The irradiation removed carriers at a rate of 2.84×10^{4} electrons/α-cm in n-type samples and 1.32×10^{4} holes/α-cm in p-type samples. Thermal recovery was studied up to 78°K. No measurable recovery was observed below 22°K. Upon warming to 78°K the irradiation-induced changes of Hall coefficient decreased by about the same fraction, 25% and 22%, respectively, in n-type and p-type samples, showing that the same fractions of donor and acceptor defects were annealed out. Experiments made with various annealing procedures, using electrical resistivity as a measure, indicated that there were two distinct regions of thermal recovery, with maximum rates of change occurring near 33°K and 67°K. The low-temperature process appears to follow a first-order reaction with a unique activation energy of about 0.02 ev. It may be due to the recombination of close interstitial-vacancy pairs. The higher-temperature process does not yield to simple analysis.

ENERGY-BAND INTERPOLATION SCHEME BASED ON A PSEUDOPOTENTIAL. James C. Phillips, Bell Telephone Laboratories, Murray Hill, New Jersey (Received July 14, 1958).

In this paper an interpolation scheme is developed that depends on only a few parameters. This is done by observing that the effective potential for electrons near the Fermi level can be split into two parts, the part due to the core, and the part due to the other valence electrons. It is assumed that for semiconductors the relative effect of the core is small, so that it can be replaced by an effective repulsive potential. In this way two-parameter pseudopotentials are constructed for diamond and Si that give good agreement with orthogonalized plane wave calculations and experiment at special points of the Brillouin zone, and also yield reasonable results for the bands at other points of the zone. Good results are obtained for Ge with a three-parameter pseudopotential. Finally, the many experimental facts that have been deduced about the band structures of Si and Ge are augmented by the results of the pseudopotential calculations to yield fairly accurate ($\delta E \lesssim 0.05$ ry) sketches of the energy bands of these crystals along the [100] and [111] directions in the neighborhood of the energy gap.

FREE CARRIER ABSORPTION IN n-TYPE Ge. R. Rosenberg and Melvin Lax, Bell Telephone Laboratories, Incorporated, Murray Hill, New Jersey (Received June 19, 1958).

The structure of the conduction band of Ge is completely taken into account in calculating the cross section in second-order Born approximation. The second-order matrix element contains first-order matrix elements of electron-photon interaction (derived for anisotropic mass) and of electron-scatterer interaction. Scatterers considered are ionized impurities and phonons belonging to all six branches of the vibrational spectrum. Deformation parameters of electron-phonon interaction are chosen to agree with other experiments. An excellent fit to high-temperature data (450°K), where impurity effects are very small, is obtained without adjusting any parameters. Contributions to the cross

section from processes involving scattering by longitudinal and transverse long-wavelength acoustic phonons are comparable; their combined effect is matched by energetic-phonon effects when $T \gtrsim 300°K$. In this temperature range, when $40\,\mu \gtrsim \lambda \gtrsim 10\,\mu$, contributions from all phonon processes are virtually indistinguishable in their temperature and wavelength (λ^2) dependence. Predicted cross sections are too low by 20-30% at 293°K, and by a factor of 2-4 at 78°K. The low-temperature results are in apparent disagreement with theoretical results of Fan, Spitzer, and Collins. It is suggested that their theory leads to an overestimate in the impurity calculation, and that impurity effects have not yet been explained.

DYNAMICS OF INTERACTING SPIN SYSTEMS. Robert T. Schumacher, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received July 3, 1958).

The dynamics of the approach of two spin systems to the same spin temperature is examined. It is found that for two spin systems in a rigid lattice interacting weakly with the lattice and with each other, differential equations governing the time rate of change of the spin temperatures can be derived. The derivation assumes the validity of the concept of spin temperature and the validity of the principle of detailed balance applied to eigenstates of the spin Hamiltonians of the systems. The spin mixing experiments of Abragam and Proctor provide evidence for the correctness of the differential equations thus derived. Additional experiments on spin-lattice relaxation times in NaBr, NaI, and NaCl are reported and interpreted in terms of the differential equations developed here. It is shown that under certain conditions the presence of a second spin system can shorten the relaxation time of a spin system even at rather high values of the external field.

PARAMAGNETIC RESONANCE STUDY OF COPPER DOPED SILVER CHLORIDE. Ronald F. Tucker, Jr.,* University of Illinois, Urbana, Illinois (Received June 19, 1958).

This paper gives the results of paramagnetic resonance studies of copper in silver chloride crystals at temperatures between 77°K and room temperature. The crystals were treated in various ways by illumination or halogenation. Resonances were observed in halogenated crystals which are interpreted as arising from Cu^{++} ions occupying sites which differ as to the presence and relative positions of associated vacancies. These spectra were observed to vary with temperature in a manner attributable to vacancy motion. The activation energy for this motion is estimated. An estimate is made of the amount of Cu-Cl bonding through an approximate linewidth analysis. Speculations are offered as to the role of the copper ion in the photolytic process in silver chloride, and further experiments are suggested.

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SURFACE TRANSPORT THEORY. Jay N. Zemel, United States Naval Ordnance Laboratory, White Oak, Silver Spring, Maryland (Received July 3, 1958).

A theory is presented for the dependence of the galvanomagnetic parameters on the surface potential of a semiconductor. The expression for the conductivity reduces to that given by Schrieffer when the magnetic field is zero. Formal equations for the magnetoconductivity and Hall coefficient are derived. By using a constant relaxation time, and a linear space charge region potential, one can obtain closed form expressions for the conductivity effective mobility and the Hall effective mobility. The Hall mobility is found to be 13% smaller than the conductivity mobility for large values of the surface potential.

ROLE OF TRAPS IN THE PHOTOELECTROMAGNETIC AND PHOTOCONDUCTIVE EFFECTS. R. N. Zitter, Chicago Midway Laboratories, The University of Chicago, Chicago, Illinois (Received June 6, 1958).

When carriers recombine through traps, the excess concentrations of mobile electrons and holes are not necessarily characterized by a single lifetime τ. Under the assumption that electrons and holes have separate lifetimes which in general are different, expressions for the steady-state photoelectromagnetic and photoconductive currents are obtained which show that in certain cases the photoelectromagnetic current is determined by a lifetime different from the one determining the photoconductive

current. The results of the photoelectromagnetic and photoconductive measurements can be used to evaluate the parameters of any particular model which might be postulated for the recombination process. However, the theoretical treatment of the photoelectromagnetic and photoconductive effects presented here is independent of such models and so can be used as a method for testing their validity.

SPIN ROTATIONAL CONSTANT OF H_2. G. E. Tauber, Department of Physics, Western Reserve University, Cleveland, Ohio (Received July 10, 1958).

The spin rotational constant of H_2 has been calculated by considering the motion of the electrons in the presence of the two nuclei which are themselves permitted to move. The interaction is described by Breit's two-body Hamiltonian suitably generalized to this problem. The wave function is obtained by the Born-Oppenheimer method of adiabatic approximation. Calculations have been carried out for Wang's electronic wave function and up to first order in the expansion parameter give for the effective magnetic field due to rotation 23.6 gauss in good agreement with the experimental result of 27.0 gauss.

GENERAL IMPACT THEORY OF PRESSURE BROADENING. Michel Baranger, Carnegie Institute of Technology, Pittsburgh, Pennsylvania, and The RAND Corporation, Santa Monica, California (Received July 8, 1958).

The work of two previous papers is extended and a theory of pressure broadening is developed which treats the perturbers quantum mechanically and allows for inelastic collisions, degeneracy, and overlapping lines. The impact approximation is used. It consists in assuming that it takes, on the average, many collisions to produce an appreciable disturbance in the wave function of the atom, and it results in an isolated line having a Lorentz shape. Validity criteria are given. When the approximation is valid, it is allowable to replace the exact, fluctuating interaction of the perturbers with the atom by a constant effective interaction. The effective interaction is expressed in terms of the one-perturber quantum-mechanical transition amplitudes on and near the energy shell and its close relationship to the scattering matrix is stressed. The calculation of the line shape in terms of the effective interaction is the same as when the perturbers move on classical paths. Results are written explicitly for isolated lines. If the interaction of the perturbers with the final state can be neglected, the shift and width are proportional to the real and imaginary part of the forward elastic scattering amplitude, respectively. By the optical theorem, the width can also be written in terms of the total cross section. When the interaction in the final state cannot be neglected, the shift and width are still given in terms of the elastic scattering amplitudes, in a slightly more complicated fashion. Finally, rules are given for taking into account rotational degeneracy of the radiating states.

STRUCTURE OF THE NUCLEAR MASS SURFACE. George A. Baker, Jr., Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received July 2, 1958).

We analyze the nuclear mass surface in terms of its observed structural properties, and develop a formula for the binding energy which is easy to use and is accurate, over all, to ± 0.54 Mev, and, for $A > 214$, to ± 0.27 Mev.

RECOIL EFFECT IN BETA DECAY AND K CAPTURE. Claude C. Bouchiat, Palmer Physical Laboratory, Princeton University, Princeton, New Jersey (Received June 25, 1958).

The study of recoil effects in β decay and K capture is extended to the case of forbidden transitions. Formulas for the angular distribution of recoil nuclei from oriented parent nuclei, the polarization of the recoil nucleus, and the angular correlation between the recoil nucleus and the following γ ray emitted in given state of circular polarization are presented. For β decay the results, in which the Coulomb effects have been taken into account only through the Fermi function, are given in closed form for nth forbidden transitions. We have considered separately ST and VA interactions. The results are quite sensitive to the choice between the two couplings. In particular it is shown that for the so called unique forbidden transitions of light nuclei, parity nonconserving effects vanish for a VA interaction, whereas they are expected to be large with an ST interaction. The study of recoil effects in K capture, which has been limited to the case of first-forbidden transitions,

provides a set of experiments which may be used to determine the helicity of the neutrino.

DELAYED NEUTRONS FROM THE SPONTANEOUS FISSION OF Cf^{252}. S. Cox, P. Fields, A. Friedman, R. Sjoblom, and A. Smith, Argonne National Laboratory, Lemont, Illinois (Received July 1, 1958).

The periods and absolute yields of delayed neutrons emitted following the spontaneous fission of Cf^{252} have been measured. Delayed neutron periods of $\tau_{(1/2)} = 0.5 \pm 0.4$, 2.0 ± 0.4 and 20.0 ± 0.5 sec with respective absolute yields of 0.35, 0.29 and 0.22% were found. The experimental results have been correlated with delayed neutron emission from other fission processes and with existing precursor systematics.

EXCITED STATES OF Cu^{64} and Cu^{66}. R. P. de Figueiredo, M. Mazari,* and W. W. Buechner, Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received July 14, 1958).

The $Cu^{63}(d, p)Cu^{64}$ and $Cu^{65}(d, p)Cu^{66}$ reactions have been investigated through studies of the proton groups arising from the deuteron bombardment of thin targets of isotopically enriched copper. The incident deuteron beam, with energies ranging from 6.00 to 6.55 Mev, was obtained from an electrostatic accelerator, and the protons were analyzed with a high-resolution magnetic spectrograph. In the region of excitation up to 3.80 Mev, sixty-five excited states in Cu^{64} and fifty-five in Cu^{66} have been measured. The Q values for the transitions to the ground states of Cu^{64} and Cu^{66} are 5.691 ± 0.008 Mev and 4.832 ± 0.008 Mev, respectively.

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SEARCH FOR ELECTRIC MONOPOLE PAIRS FROM THE 3.82-Mev STATE OF Ca^{48}. K. E. Eklund and R. D. Bent,* Columbia University, New York, New York (Received July 15, 1958).

A search was made for 3.82-Mev electric monopole pairs from the reaction $Ca^{48}(p, p')Ca^{48}$* using a $CaCO_3$ (35% Ca^{40}, 62% Ca^{48}) target, 6.0-Mev protons, and an intermediate-image spectrometer. No 3.82-Mev pairs from Ca^{48} were observed with an intensity as great as 0.3% that of the 3.35-Mev electric monopole pairs from Ca^{40}. This result suggests that the 3.72-Mev state of Ca^{48} decays predominantly by gamma emission. This could occur if the 3.82-Mev state has spin >0, or if there are lower excited states in Ca^{48}.

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RADIATIONS OF Xe^{127}. R. N. Forrest and H. T. Easterday, Department of Physics, University of Oregon, Eugene, Oregon (Received July 1, 1958).

The radiations of Xe^{127} have been examined. The energies of previously observed gamma rays were found to be 58 ± 1, 146 ± 0.5, 173.0 ± 0.5, 204.5 ± 0.5 and 377.5 ± 0.5 kev. Conversion-electron intensities relative to gamma-ray intensities and internal conversion coefficients were measured. The experiment indicates that electron capture to the ground state is suppressed. Spin and parity assignments are proposed for the 204- and 377-kev excited states of I^{127}.

INVERSE BETA DECAY AND THE TWO COMPONENT NEUTRINO. R. W. King, Purdue University, Lafayette, Indiana, and J. F. Perkins, Lockheed Aircraft Corporation, Marietta, Georgia (Received July 14, 1958).

Several procedures for calculating the average cross-section per anti-neutrino from U^{235} fission are given to test the predictions of the two-component neutrino theory. A <u>firm</u> lower limit of $\bar{\sigma}_p > 7 \times 10^{-44}\,cm^2$ is deduced from the known decays of the fission products. Three different procedures, if weighted equally, give a "best value" of $\bar{\sigma}_p = 14 \times 10^{-44}\,cm^2$ to be compared with the recently increased experimental value $\bar{\sigma}_p = (11 \pm 4) \times 10^{-44}\,cm^2$. It is concluded that predictions of the two-component neutrino theory are in accord with the experimental results on inverse beta decay.

SEARCH FOR ELECTRIC MONOPOLE PAIRS FROM THE 7.6-Mev STATE OF C^{12}. T. H. Kruse and R. D. Bent,* Columbia University, New York, New York (Received July 15, 1958).

A search was made for weak 7.6-Mev electric monopole pairs from the reaction $Be^9(\alpha, n)C^{12}$*

using a thin target, 5.7-Mev alpha particles, and a scintillation pair spectrometer. No 7.6-Mev pairs were seen with an intensity as great as 1.6×10^{-5} that of 4.43-Mev gamma rays. This result gives a limit of $\Gamma_{\pm e}\Gamma_{\text{total}} < 1.3\times10^{-4}$ for the ratio of the partial width for pair emission to the total width of the 7.6-Mev state. If an estimated alpha-particle width of 0.5 ev is taken for the total width of the 7.6-Mev state, then an upper limit of $\Gamma_{\pm e} < 7\times10^{-5}$ ev is obtained for the partial width for pair emission of the 7.6-Mev state.

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RADIO-FREQUENCY ORIENTATION OF Sb^{122}. Francis M. Pipkin, Lyman Laboratory, Harvard University, Cambridge, Massachusetts (Received June 16, 1958).

Radioactive Sb^{122} donors in a doped silicon crystal have been oriented by saturation of forbidden transitions and by the Overhauser method. Double-resonance experiments in which an electron and a nuclear transition were saturated simultaneously were used to determine the Sb^{122} hyperfine splitting. A short discussion is given of how the nuclear relaxation mechanisms affect the double-resonance orientation signals. The hyperfine splitting of Sb^{122} has been found to be -132.59 ± 0.10 Mc/sec and its spin 2. The gyromagnetic ratio is -0.952 ± 0.05.

DIFFERENTIAL CROSS SECTIONS FOR PHOTODISINTEGRATION OF THE DEUTERON AT FAR FORWARD AND BACKWARD ANGLES. C. A. Tatro,* T. R. Palfrey, Jr., R. M. Whaley,† and R. O. Haxby,‡ Purdue University, Lafayette, Indiana (Received July 7, 1958).

Direct measurements of differential cross sections for photodisintegration of the deuteron have been made at center-of-mass angles of 11°, 100°, and 176° using photons with energies near 200 Mev.

The cross sections at 176° were found to be 3.8 ± 0.6 μb/sterad and 5.2 ± 0.6 μb/sterad at a mean photon energy of 190 Mev and 238 Mev respectively. These cross sections are higher than would be predicted by extrapolation from the angular distributions measured by other workers. The cross sections at the other two angles were in satisfactory agreement with previous measurements or extrapolations of measurements.

The behavior of the differential cross sections at the far backward angle is taken as additional evidence of the close relation between photomeson production and photodisintegration of the deuteron in this energy region.

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ANGULAR DISTRIBUTIONS OF ELASTICALLY AND INELASTICALLY SCATTERED PROTONS FROM INDIUM. R. D. Sharp * and W. W. Beuchner, Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts (Received July 14, 1958).

Thin targets of natural indium were bombarded with 7.04-Mev protons and the scattered particles were analyzed with a magnetic spectrograph. Inelastically scattered protons were observed corresponding to excited states in In^{115} at 1.078, 1.135, 1.292, and 1.982 Mev. Measurements were made on the variation of the absolute cross sections for elastic and inelastic scattering, and it is concluded that a direct interaction process is important in the excitation of these states.

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SEMICLASSICAL TREATMENT OF DIRECT NUCLEAR REACTIONS. S. T. Butler, N. Austern, and C. Pearson, The F.B.S. Falkiner Nuclear Research and Adolph Basser Computing Laboratories, School of Physics, The University of Sydney, Sydney, N.S.W., Australia (Received June 26, 1958).

A semiclassical method is employed to discuss the angular distributions of direct nuclear reactions. The method considers the incident and outgoing particles to be described by rays which follow classical paths; these rays can be refracted, reflected and absorbed by a "clouded crystal ball" nucleus. Any given incident ray is considered to have a certain probability of suffering a major "scattering" at each point within a nucleus, this "scattering" not only changing the direction of the ray but also its wavelength. A condition of angular momentum conservation is imposed on this direct scattering event, such that the angular momentum change in this pro-

cess is equal to that required by the direct reaction under consideration.

On this picture it is seen that the oscillations in the angular distributions which often occur with direct reactions arise as a result of interference between direct scatterings in different regions of the nucleus. Thus these oscillations are to be likened to the interference maxima and minima which arise when light is scattered, for example from a soap film. Effects due to compound-nucleus absorption and of refraction and reflection of the incident and outgoing rays can be discussed fairly simply on this model.

An investigation of the basis of the semiclassical method indicates that it should be reliable over a wide range of interesting conditions.

GAMMA-RAY ABSORPTION IN CARBON. Milo M. Wolff and William E. Stephens, Physics Department, University of Pennsylvania, Philadelphia, Pennsylvania (Received June 9, 1958).

The total absorption in carbon of the gamma rays from the $T^3(p,\gamma)He^4$ reaction was measured as a function of proton energy and hence gamma-ray energy. As the gamma-ray energy was varied from 20.3 to 20.8 Mev, the total absorption cross section in carbon varied smoothly from 315 ± 5 to 324 ± 6 millibarns. The resolution was better than 70 kev and the sensitivity sufficient to detect narrow resonances of more than 0.8 Mev-millibarns integrated cross section, but no structure was observed although photonuclear resonances have been reported in this region.

FIRST EXCITED STATE OF B^{11} AND SPIN-FLIP STRIPPING. J. E. Bowcock, Department of Physics, University of Wisconsin, Madison, Wisconsin (Received July 14, 1958).

A theoretical treatment is presented of the anomalous angular distribution in the $B^{10}(d,p)B^{11}$ reaction to the first excited state of the final nucleus. The experimental distribution is characteristic of the orbital angular momentum of the captured neutron being 1. However, the known spins and parities of the target and final-state nuclei do not admit this value. By coupling the spin of the stripped proton to the spin of the system comprising target nucleus plus neutron, an extra unit of angular momentum may be imparted to this system, allowing the final state to be reached. The angular distribution from such a mechanism is calculated and fits rather well the experimental curves.

SHAPE OF Pr^{143} BETA SPECTRUM. J. H. Hamilton, L. M. Langer, R. L. Robinson, * and W. G. Smith, † Physics Department, Indiana University, Bloomington, Indiana (Received July 10, 1958).

Small deviations from the statistical shape which has previously been reported for Pr^{143} might be expected from a theoretical prediction for such a once-forbidden nonunique transition. The spectrum was investigated with greater precision and in greater detail than in past studies, with magnetic and scintillation spectrometers. The spectrum is found to exhibit small deviations from a statistical shape. The spectrum can be fitted with a once-forbidden, nonunique shape factor.

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LIFETIMES OF THE FIRST EXCITED STATES OF F^{17} AND B^{10}. R. E. Holland, Frank J. Lynch, and S. S. Hanna, Argonne National Laboratory, Lemont, Illinois (Received July 2, 1958).

The mean life of the first excited state of F^{17} has been measured using a pulsed-beam technique. The mean life is definitely <0.5 mμsec. The result obtained is (0.35 ± 0.15) mμsec with the upper limit more firmly established than the lower. A value of 495 ± 15 kev was obtained for the excitation energy in F^{17}. A measurement of the mean life of the first excited state of B^{10} gave a value of (0.90 ± 0.1) mμsec.

$Li^6(n,t)He^4$ CROSS SECTION FOR 12.5- TO 18.3-Mev NEUTRONS. Bernard D. Kern and William E. Kreger, U. S. Naval Radiological Defense Laboratory, San Francisco, California (Received June 23, 1958).

The cross section for the $Li^6(n,t)He^4$ reaction has been measured for 12.5- to 18.3-Mev neutrons. The neutrons were obtained from the $T(d,n)He^4$ reaction and their flux density was determined by counting the recoil He^4 particles. A $Li^6I(Eu)$ scintillation crystal $1\frac{1}{2}$ inches in dia-

meter by $\frac{1}{2}$ inch thick served as both the Li^6 sample and the detector for the reaction products. The cross section is nearly linear from 34.3 mb at 12.5 Mev to 17.6 mb at 18.3 Mev. It is 28.1 ± 1.6 mb at 14.2 ± 0.2 Mev.

MEASUREMENTS ON THE v/c DEPENDENCE OF THE BETA-CIRCULARLY POLARIZED GAMMA CORRELATION IN Co^{60} USING A MAGNETIC-LENS SPECTROMETER. L. A. Page*, B.-G. Pettersson, and T. Lindqvist, Institute of Physics, University of Uppsala, Uppsala, Sweden (Received July 7, 1958).

Measurements have been made of the circular polarization of Co^{60} gamma rays in coincidence with beta rays whose momentum is defined by a magnetic lens spectrometer. The gamma rays are analyzed by a transmission-type Compton analyzer of new construction (toroidal geometry). At nominal 180° between beta and gamma, and accepting both of the gamma-ray lines, the degree of circular polarization is generally higher than the expected + 0.33 (v/c) and markedly so for the smaller speeds. Proceeding downward in v/c from 0.71 to 0.33, there is no indication that the polarization falls off as fast as would be given by a "v/c law."

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APPLICATION OF PERTURBATION METHODS TO THE THEORY OF NUCLEAR MATTER. D. J. Thouless, Laboratory of Nuclear Studies, Cornell University, Ithaca, New York (Received June 19, 1958).

A generalized perturbation theory is developed in such a way that it can be applied to a many-body problem with strong forces between the particles. The Brueckner expression for the energy is shown to be the first-order term in a particular case of this expansion. Some of the higher-order terms in the expansion are studied, and the importance of self-consistency in the energy denominator of Brueckner's equation and of the use of the exclusion principle in intermediate states is assessed. A possible simplification of the methods used is suggested, which involves solving the Brueckner equation for the hard core, and using normal perturbation theory for the attractive part of the potential. The methods developed are used to analyze some details of previously published calculations.

The lack of equality between the Fermi energy and the binding energy in the nuclear matter calculations shows that there must be a rearrangement energy. A simple formula for the rearrangement energy is derived, and its importance for single-particle excited states, such as occur in the optical model, is shown. The relation between the rearrangement energy and the departure of the system from a degenerate Fermi gas state is shown. The effect of the rearrangement energy on the ground state energy is indirect, but it is as important as the self-consistency condition. The rearrangement energy seems to come mainly from the hard core, and simple numerical estimates of the rearrangement energy from a hard-core potential show that it is somewhat less than 16 Mev at the Fermi surface. The ground state energy is reduced by perhaps 1 Mev. There seems to be a discrepancy between the calculated and observed energy dependence of the real part of the optical model potential.

PHOTOPRODUCTION OF K^+ MESONS IN HYDROGEN. P. L. Donoho* and R. L. Walker, California Institute of Technology, Pasadena, California (Received June 19, 1958; revised manuscript received July 30, 1958).

K^+ mesons produced in a liquid hydrogen target bombarded by the 1100-Mev bremsstrahlung of the California Institute of Technology synchrotron have been observed. It has been found that the K^+ mesons are produced in association with Λ^0 hyperons, in accordance with the law of associated production of strange particles. The K^+ mesons were momentum-analyzed in a magnetic spectrometer and identified by their energy loss in three scintillation counters, the very large background due to pions and protons being virtually eliminated by means of time-of-flight discrimination. The differential cross section for the reaction $\gamma + p \rightarrow K^+ + \Lambda^0$ has been measured at photon energies of 960 Mev, 1000 Mev, and 1060 Mev at various K^+-meson laboratory angles between 15 degrees and 45 degrees. This cross section shows little variation with photon energy between 960 Mev and 1060 Mev or with center-of-momentum angle over the range investigated.

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LEPTONIC DECAY MODES OF THE HYPERONS. F. Eisler, R. Plano, A. Prodell, N. Samios, M. Schwartz, and J. Steinberger, Nevis Cyclotron Laboratories, Columbia University, Irvington-on-Hudson, New York, and Brookhaven National Laboratory, Upton, New York, and M. Conversi, P. Franzini, I. Manelli, R. Stangelo, and V. Silvestrini, Istituto di Fisica, Pisa, Italy (Received July 2, 1958).

We have searched for the leptonic decay of the Λ° and Σ^-. The sensitivity of the experiment was such that 5-6 events should have been found according to the predictions of the "Universal" V-A model of β decay. No examples of leptonic decay were observed

π^+-p INTERACTIONS IN THE ENERGY RANGE AROUND 500 Mev. M. E. Blevins, M. M. Block, and J. Leitner, Duke University, Durham, North Carolina (Received January 13, 1958).

A hydrogen-filled diffusion cloud chamber investigation of π^+-p interactions in the energy range around 500 Mev is reported. The elastic scattering angular distribution, based on a total of 180 events, is analyzed using only s and p waves, and also using s and p waves with a d-wave interference term. Phase shifts are presented and ambiguities are discussed. The d-wave contribution is small. The elastic cross section is used to compute the real part of the forward coherent scattering amplitude. The latter is in good agreement with that predicted by the dispersion relations. Inelastic production sets in at about 300 Mev. Eighteen inelastic events were found; 7 were $p+0$, 7 were $n++$, and 4 were unidentifiable. Inelastic processes are discussed in terms of various models.

HIGH-ENERGY ELECTRON SCATTERING FROM LITHIUM-6. George R. Burleson and R. Hofstadter, Department of Physics and High-Energy Physics Laboratory, Stanford University, Stanford, California (Received July 7, 1958).

The Li^6 nucleus has been studied at 426 Mev by electron scattering methods between laboratory angles of 33° and 57.5°. Absolute values of the Li^6 cross sections have been obtained by comparison with scattering measurements made on the proton at 426 Mev and 40°. Unique values of the experimental form factors are given in tabular form. The results are analyzed in terms of various possible charge distributions. Several examples of models showing a good fit are presented and appropriate radius and skin thickness values are given. Several unsuccessful models are also discussed.

INTERNAL PAIRS IN THE CAPTURE OF π^- ON PROTONS. Ronald M. Rockmore and John G. Taylor, Institute for Advanced Study, Princeton, New Jersey (Received April 15, 1958).

The contribution from nucleon structure to the process $\pi^- + p \rightarrow n + e^- + e^+$ is calculated. The dispersion relations for photoproduction of π mesons by virtual photons, together with crossing symmetry, are used. The nucleon structure contribution is found to be of the order of 2% and of the same size as the contribution from longitudinal polarization. The effect cannot be detected from present data. A 25% discrepancy between theory and experiment, which results from using the more reliable Panofsky ratio 1.8, still persists.

PION-HYPERON SCATTERING AND K^--p REACTIONS. Marc Ross, Physics Department, Indiana University, Bloomington, Indiana (Received July 7, 1958).

A brief survey is made of the consequences of the universal pion-baryon interaction on production of hyperons with pions. In particular, a pion-hyperon resonant scattering state similar to the $p_{3/2}$ $T=3/2$ pion nucleon state should exist. Possible effects of this state are examined. It is found that the large low-energy K^--p cross sections can not be associated with it. Other experiments are suggested in order to search for this state, especially K^--p reactions at higher energy and pion production in hyperon nucleon scattering.

NEUTRAL VECTOR MESON AND NUCLEON FORM FACTORS AND MAGNETIC MOMENTS. Robert W. Huff, The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois (Received June 16, 1958).

The existence of the heavy neutral vector meson recently suggested by Nambu is studied as a possible explanation of two phenomena: (1) the small charge rms radius of the neutron, and (2) the nucleon anomalous magnetic moments. The

types and strengths of its interactions required to account for these phenomena are then used to predict the modes and rates of its decay as a function of its mass. The possibility of its detection by observation of these decay products is briefly discussed. Results are inconclusive.

A GENERALIZED REACTION MATRIX APPROACH TO THE THEORY OF THE INFINITE MEDIUM OF FERMIONS. Richard Prange and Abraham Klein, University of Pennsylvania, Philadelphia, Pennsylvania (Received June 30, 1958).

A closed set of coupled, nonlinear integral equations are obtained which completely describe a system of Fermi particles. The equations define the one- and two-particle Green's functions which are completely analogous to those used in the field theory of fundamental particles. Special boundary conditions appropriate to the infinite medium of fermions are employed. Approximations to these equations are made which lead to a generalized Brueckner reaction-matrix formalism in the case of nuclear matter and which lead to an expression for the correlation energy in the case of the electron gas. The approximations for the system of nuclear matter are of three types: ones which neglect the simultaneous interaction of three or more nucleons, the effective interactions of the vacant states below the Fermi sea, and the self-energy interactions with excited nucleons. The relationship between the reaction matrix and the expectation value of the actual potential energy is discussed. It is found that the energy levels given by the Brueckner formalism are nearly those given by a consideration of the actual energy, thus indicating that the shell-model energy levels for closed-shell-plus-one nuclei should be almost correct. A formula for the momentum distribution of nucleons in nuclear matter is obtained.

PERTURBATION THEORY FOR THE INFINITE MEDIUM OF FERMIONS. Abraham Klein and Richard Prange, University of Pennsylvania, Philadelphia, Pennsylvania (Received June 30, 1958).

The ground-state energy and low-energy excitations of single-particle character of an infinite medium of Fermions are discussed with the aid of time-dependent Green's functions, which are convenient generalizations of the exact particle correlation functions in the ground state of N particles. The power series development for the one- and two-particle functions, under the restriction to two-body forces, is derived and described by means of Feynman diagrams. The derivation of the linked-cluster expansion for the energy then follows immediately. The equivalence to previous versions is established. The one-particle function is examined in particular detail, and it is shown that the poles of its space-time Fourier transform studied as a function of the energy variable, for fixed momentum, determine the $(N+1)$-particle and $(N-1)$-particle excited states which have single-particle character. For a reasonable assumption about the full spectrum of excited states, it is found that for the interacting system single-particle excitations with a real energy occur only at the Fermi momentum. It is pointed out that the corresponding energy, termed the perturbed Fermi energy, equals the binding energy per particle in the ground state of N particles for a saturating system at equilbrium density.

It is shown, finally, that the entire structure of the theory may be carried over to the case of finite temperature, requiring only a redefinition of the Green's functions. The analogy is constructed from a discussion of the internal energy.

NEW TECHNIQUES IN THE LAMB SHIFT CALCULATION. H. M. Fried* and D. R. Yennie,† Stanford University, Stanford, California (Received July 10, 1958).

Based on the analogy between the calculations of radiative corrections to electron scattering and the Lamb shift, a new procedure applicable to bound-state self-energy problems is developed, wherein the electron propagator is expanded in powers of the external potential. In the resulting sequence, a change in the gauge of the virtual photon field conveniently removes from each term spurious lower order contributions and yields a new and considerably simpler sequence; the expectation value of the first two terms of the latter is shown to account for the major portion of the Bethe logarithm. A simple method is developed to sum all the $\alpha(Z\alpha)^4\mu$ dependence, and the result is the lowest order Lamb shift formula. The ease of the calculation, as well as that involved in obtaining the relativistic level shift correction of order $\alpha(Z\alpha)^5\mu$ (not given in the present paper), indicates that the

method may find application in the calculation of further higher-order effects.

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MESON EXCHANGE EFFECTS IN TWO-NUCLEON STATES. R. E. Cutkosky, Carnegie Institute of Technology, Pittsburgh, Pennsylvania (Received June 19, 1958).

It is suggested that the most fruitful study of meson exchange effects requires a unified treatment of all processes in which they occur. A new representation of the eigenstates of the two-nucleon system, based on the Heitler-London method, is proposed as being particularly useful for this purpose. Heitler-London states have the property that they become equal to the exact eigenstates when the two nucleons are far apart. Matrix elements between Heitler-London states can be expressed in terms of the properties of isolated nucleons, by means of expansions in the number of exchanged mesons. The Heitler-London method affords a mathematically precise method of incorporating phenomenological isobar effects into a field-theoretical model. Most of the discussion of the details of the formalism is confined to the fixed-source model, but in the final part of the paper the application to a more general model is discussed briefly.